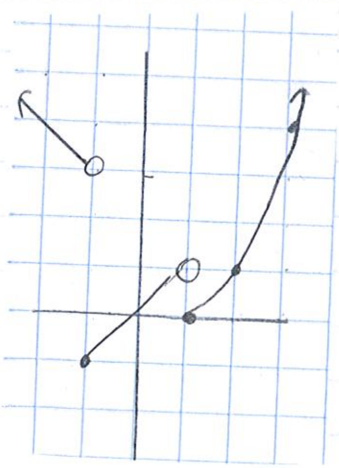
Part II Limits and Derivatives

1\.
a. -1
b. 1
c. DNE $L \neq R$
d. 1
e. 1
f. 2
g. DNE $L \neq R$
h. 2
i. DNE oscillating
j. 2
k. DNE

2\.
a -1
b -2
c DNE $L \neq R$
d 2
e 0
f DNE $L \neq R$
g 1
h 3

3\.
a DNE unbound
b DNE unbound
c DNE $-\infty$
d DNE $+\infty$
e $x = -3$
$x = 2$
$x = 5$

4\.



limit exists for all x values except $x \neq -1, 1$

5\. a. $+\infty$ b. $-\infty$ c. DNE unbound $+\infty$ d. DNE unbound $-\infty$ e. $-\infty$

6\.
a. $-3+8 = 5$
b. $(-3)^2 = 9$
c. $\sqrt[3]{8} = 2$
d. $\frac{1}{-3} = -\frac{1}{3}$
e. $\frac{-3}{8}$
f. $\frac{0}{-3} = 0$
g. $\frac{-3}{0}$ = DNE can't divide by zero
h. $\frac{2(-3)}{8-(-3)} = \frac{-6}{11}$

7. a. $3(-2)^4+2(-2)^2-(-2)+1 = \underline{\underline{59}}$

b. $\dfrac{2(2)^2+1}{(2)^2+6(2)-4} = \dfrac{9}{12} = \underline{\underline{\dfrac{3}{4}}}$

c. $((-1)^2+1)^3(-1+3)^5$
$2^3 \cdot 2^5 = 2^8 = \underline{\underline{256}}$

d. $\left(\dfrac{1+3}{1+4+3}\right)^3 = \left(\dfrac{4}{8}\right)^3 = \left(\dfrac{1}{2}\right)^3 = \underline{\underline{\dfrac{1}{8}}}$

e. $\sqrt{16-4^2}$
$\sqrt{0} = \underline{\underline{0}}$

8. a. $\lim_{t\to -3} \dfrac{\cancel{(t+3)}(t-3)}{(2t+1)\cancel{(t+3)}}$

$\lim_{t\to -3} \dfrac{t-3}{2t+1}$

$\dfrac{-3-3}{-6+1} = \dfrac{-6}{-5} = \boxed{\dfrac{6}{5}}$

b. $\lim_{h\to 0} \dfrac{\cancel{16}+8h+h^2-\cancel{16}}{h}$

$\lim_{h\to 0} \dfrac{\cancel{h}(8+h)}{\cancel{h}}$

$\boxed{8}$

c. $\lim_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7} \cdot \dfrac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$

$\lim_{x\to 7} \dfrac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$

$\lim_{x\to 7} \dfrac{1}{\sqrt{x+2}+3}$

$\dfrac{1}{\sqrt{9}+3} = \boxed{\dfrac{1}{6}}$

d. $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} \left(\dfrac{4x}{4x}\right)$

$\lim_{x\to -4} \dfrac{\frac{x+4}{4x}}{4+x}$

$\lim_{x\to -4} \dfrac{\cancel{x+4}}{4x} \cdot \dfrac{1}{\cancel{4+x}}$

$\lim_{x\to -4} \dfrac{1}{4x}$

$\boxed{-\dfrac{1}{16}}$

9. a) $\lim_{x\to 2^-} f(x) = 4-(2)^2 = \underline{\underline{0}}$ b. $\lim_{x\to 2^+} 2-2 = \underline{0}$ c. yes at $\underline{0}$

10. $f(4) = 4^2 + \sqrt{7-4}$
$16 + \sqrt{3}$

f(x) is continuous for all $x \le 7$
4 is in domain

11. $1+0^2 = 1$ $2-0=2$ $2-2=0$ $(2-2)^2=0$

NOT continuous at 0

yes continuous

$\boxed{\text{discontinuous when } x=0}$

12. $cx+1 = cx^2-1$

$c(3)+1 = c(3)^2-1$

$3c+1 = 9c-1$

$2 = 6c$

$\boxed{\frac{1}{3} = c}$

13. a. $\lim_{x\to1} \frac{3-2x+4x^2-(3-2+4)}{x-1}$

$\lim_{x\to1} \frac{3-2x+4x^2-5}{x-1}$

$\lim_{x\to1} \frac{4x^2-2x-2}{x-1} \rightarrow \frac{2(2x^2-x-1)}{x-1}$

$\lim_{x\to1} \frac{2(2x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$2(2+1)$

$\boxed{f'(1) = 6}$

b. $\lim_{x\to7} \frac{\frac{1}{\sqrt{x+2}} - \frac{1}{3}}{x-7} \quad \left(\frac{3\sqrt{x+2}}{3\sqrt{x+2}}\right)$

$\lim_{x\to7} \frac{\frac{3-\sqrt{x+2}}{3\sqrt{x+2}}}{x-7}$

$\lim_{x\to7} \frac{3-\sqrt{x+2}}{3(x-7)(\sqrt{x+2})} \cdot \frac{3+\sqrt{x+2}}{3+\sqrt{x+2}}$

$\lim_{x\to7} \frac{9-(x+2)}{3(x-7)(\sqrt{x+2})(3+\sqrt{x+2})}$

$\lim_{x\to7} \frac{\cancel{7-x}}{3\cancel{(x-7)}(\sqrt{x+2})(3+\sqrt{x+2})}$

$\lim_{x\to7} \frac{-1}{3(\sqrt{x+2})(3+\sqrt{x+2})}$

$\frac{-1}{3(3)(6)}$

$\boxed{f'(7) = -1/54}$

14. $g(x) = 1-x^3$

$g'(x) = -3x^2$

$g'(0) = 0$

$g'(0) = 0 \quad (0,1)$

$\boxed{y = 1}$

15. $f(x) = 3x^2-5x$

$f'(x) = 6x-5$

$f'(2) = 12-5$

$f'(2) = 7$

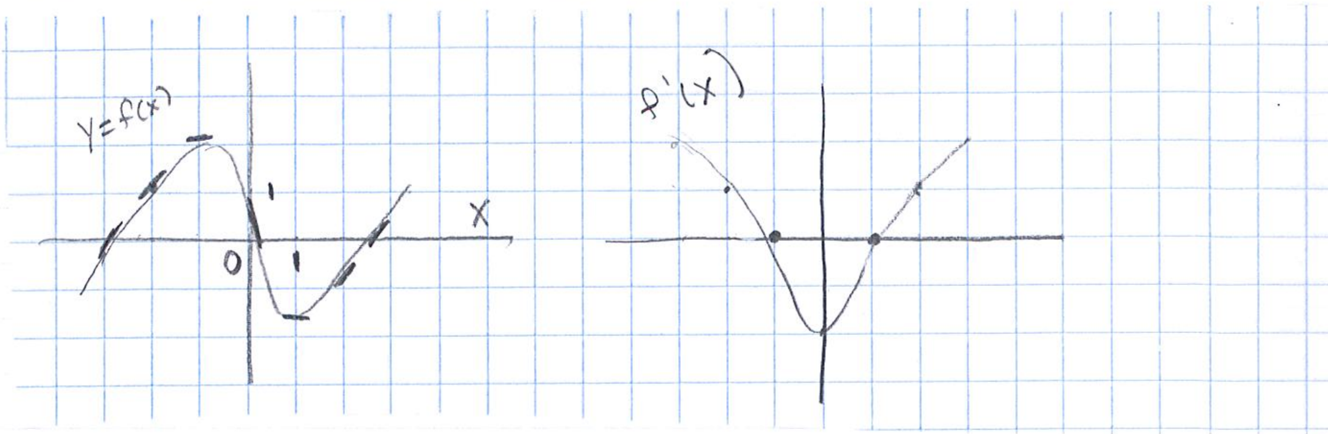
$f'(2) = 7 \quad (2,2)$

$y-2 = 7(x-2)$

$y-2 = 7x-14$

$\boxed{y = 7x-12}$

16. a) $f'(-3) \approx 2$ b) $f'(-2) \approx 1$ c) $f'(-1) \approx 0$ d) $f'(0) \approx -2$
e) $f'(1) \approx 0$ f) $f'(2) \approx 1$ g) $f'(3) \approx 2$



17. a) II b) IV
c) I d) III

18 a) discontinuous when
$x = -2$ hole
$x = 0$ asymptote
$x = 5$ jump

b) not differentiable
$x = -2, 0, 5$ because NOT continuous
$x = 2$ cusp (corner)

19. a. $f'(x) = 0$
b. $f'(x) = 0$
c. $f'(x) = 5$
d. $F'(x) = -40x^9$
e. $g'(x) = 40x^7 - 10x^4$
f. $f(t) = \frac{1}{4}t^4 + 2$
$f'(t) = t^3$

g. $f'(t) = 3t^5 - 12t^3 + 1$
h. $V'(r) = 4\pi r^2$
i. $R'(t) = -3t^{-8/5}$
j. $Y'(t) = -54t^{-10}$
k. $R(x) = \sqrt{10}\,x^{-7}$
$R'(x) = -7\sqrt{10}\,x^{-8}$
$R'(x) = \frac{-7\sqrt{10}}{x^8}$

l. $f(t) = t^{1/2} - t^{-1/2}$
$f'(t) = \frac{1}{2}t^{-1/2} + \frac{1}{2}t^{-3/2}$
$f'(t) = \frac{1}{2\sqrt{t}} + \frac{1}{2\sqrt{t^3}}$
m. $y' = -\frac{2}{5}x^{-7/5}$
n. $y = x^{1/3}$
$y' = \frac{1}{3}x^{-2/3}$
$y' = \frac{1}{3\sqrt[3]{x^2}}$
o. $y' = 8\pi r$
p. $g(u) = \sqrt{2}\,u + (3u)^{\frac{1}{2}}$
$g'(u) = \sqrt{2} + \frac{3}{2}u^{-1/2} \rightarrow \sqrt{2} + \frac{3}{2\sqrt{u}}$
q. $y = x + x^{-1}$
$y' = 1 - 1x^{-2} \rightarrow y' = 1 - \frac{1}{x^2}$

20. a. power
$$y = x^{3/2} - x^{1/2}$$
$$y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$$
$$y' = \frac{3\sqrt{x}}{2} - \frac{1}{2\sqrt{x}}$$

b. Quotient
$$g(x) = \frac{3x-1}{2x+1}$$
$$g'(x) = \frac{(2x+1)3 - (3x-1)(2)}{(2x+1)^2}$$
$$g'(x) = \frac{6x+3-6x+2}{(2x+1)^2}$$
$$g'(x) = \frac{5}{(2x+1)^2}$$

c. Quotient
$$f(t) = \frac{2t}{4+t^2}$$
$$f'(t) = \frac{(4+t^2)(2) - 2t(2t)}{(4+t^2)^2}$$
$$f'(t) = \frac{8+2t^2-4t^2}{(4+t^2)^2}$$
$$f'(t) = \frac{8-2t^2}{(4+t^2)^2}$$

d. Quotient
$$y = \frac{x^{1/2}-1}{x^{1/2}+1}$$
$$y' = \frac{(x^{1/2}+1)(\frac{1}{2}x^{-1/2}) - (x^{1/2}-1)(\frac{1}{2}x^{-1/2})}{(x^{1/2}+1)^2}$$
$$y' = \frac{\frac{x^{1/2}+1}{2x^{1/2}} - \frac{x^{1/2}-1}{2x^{1/2}}}{(x^{1/2}+1)^2}$$
$$y' = \frac{\frac{x^{1/2}+1-x^{1/2}+1}{2x^{1/2}}}{(x^{1/2}+1)^2}$$
$$y' = \frac{\frac{\cancel{2}}{\cancel{2}x^{1/2}}}{(x^{1/2}+1)^2}$$
$$y' = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$

e. power
$$y' = 2ax + b$$

f.
$$y = A + Bx^{-1} + Cx^{-2}$$
$$y' = -Bx^{-2} - 2Cx^{-3}$$
$$y' = \frac{-B}{x^2} - \frac{2C}{x^3}$$

21. a.
$$y = \frac{2x}{x+1}$$
Quotient
$$y' = \frac{(x+1)(2) - 2x(1)}{(x+1)^2}$$
$$y' = \frac{\cancel{2x}+2-\cancel{2x}}{(x+1)^2}$$
$$y' = \frac{2}{(x+1)^2}$$
$$y'(1) = \frac{2}{4} = \frac{1}{2}$$

Tangent line

$y'(1) = \frac{1}{2}$ (1, 1)
$$y - 1 = \frac{1}{2}(x-1)$$
$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$
$$y = \frac{1}{2}x + \frac{1}{2}$$

perpendicular

slope -2 (1, 1)
$$y - 1 = -2(x-1)$$
$$y - 1 = -2x + 2$$
$$y = -2x + 3$$

21. b. $y = \dfrac{x^{\frac{1}{2}}}{x+1}$

Quotient

$y' = \dfrac{(x+1)\frac{1}{2}x^{-1/2} - x^{1/2}(1)}{(x+1)^2}$

$y' = \dfrac{\frac{x+1}{2\sqrt{x}} - x^{1/2}}{(x+1)^2} \left(\dfrac{2x^{1/2}}{2x^{1/2}}\right)$

$y' = \dfrac{\frac{x+1-2x}{2\sqrt{x}}}{(x+1)^2}$

$y' = \dfrac{1-x}{2\sqrt{x}\,(x+1)^2}$

$y'(4) = \dfrac{1-4}{4(5)^2} = \dfrac{-3}{100}$

Tangent Line

$y'(4) = \dfrac{-3}{100} = -.03 \quad (4, .4)$

$y - .4 = -.03(x-4)$

$y - .4 = -.03x + .12$

$y = -.03x + .52$

Perpendicular

slope $= \dfrac{100}{3} \quad (4, .4)$

$y - .4 = \dfrac{100}{3}(x-4)$

$y - .4 = \dfrac{100}{3}x - \dfrac{400}{3}$

$y - \dfrac{2}{5} = \dfrac{100}{3}x - \dfrac{400}{3}$

$- \dfrac{2000}{15}$

$+\dfrac{2}{5} \qquad + \dfrac{6}{15}$

$y = \dfrac{100}{3}x - \dfrac{1994}{15}$

c. $y = x + x^{1/2}$

$y' = 1 + \frac{1}{2}x^{-1/2}$

$y' = 1 + \dfrac{1}{2\sqrt{x}}$

$y'(1) = 1 + \dfrac{1}{2}$

$y'(1) = \dfrac{3}{2}$

Tangent Line

$y'(1) = 3/2 \quad (1, 2)$

$y - 2 = 3/2(x-1)$

$y - 2 = 3/2x - 3/2$

$+2 \qquad +4/2$

$y = 3/2x + 1/2$

Perpendicular

slope: $-2/3 \quad (1, 2)$

$y - 2 = -\dfrac{2}{3}(x-1)$

$y - 2 = -\dfrac{2}{3}x + \dfrac{2}{3}$

$+2 \qquad + \dfrac{6}{3}$

$y - 2 = -\dfrac{2}{3}x + \dfrac{8}{3}$

d. $y = (1+2x)^2$

chain

$y' = 2(1+2x)'(2)$

$y' = 4(1+2x)$

$y' = 4 + 8x$

$y'(1) = 4 + 8 = 12$

OR $y = 1 + 4x + 4x^2$

$y' = 4 + 8x$

Tangent

$y'(1) = 12 \quad (1, 9)$

$y - 9 = 12(x-1)$

$y - 9 = 12x - 12$

$y = 12x - 3$

Perpendicular

slope $-\dfrac{1}{12} \quad (1, 9)$

$y - 9 = -\dfrac{1}{12}(x-1)$

$y - 9 = -\dfrac{1}{12}x + \dfrac{1}{12}$

$+9 \qquad \dfrac{108}{12}$

$y = -\dfrac{1}{12}x + \dfrac{109}{12}$

22. a. $y = x\,g(x)$

$$y' = x\,g'(x) + g(x)(1)$$

$$y = x\,g'(x) + g(x)$$

b. $y = \frac{x}{g(x)}$

$$y' = \frac{g(x)(1) - x\,g'(x)}{(g(x))^2}$$

$$y' = \frac{g(x) - x\,g'(x)}{(g(x))^2}$$

c. $y = \frac{g(x)}{x}$

$$y' = \frac{x\,g'(x) - g(x)(1)}{x^2}$$

$$y' = \frac{x\,g'(x) - g(x)}{x^2}$$

23. a. $y = x - 3\sin x$

$y' = 1 - 3\cos x$

b. $y = x\sin x$

$y' = x(\cos x) + \sin x(1)$

$y' = x\cos x + \sin x$

c. $y = \sin x + 10\tan x$

$y' = \cos x + 10\sec^2 x$

d. $y = 2\csc x + 5\cos x$

$y = -2\csc x\cot x - 5\sin x$

e. $g(t) = 4\sec t + \tan t$

$g'(t) = 4\sec t\tan t + \sec^2 t$

f. $y = \frac{x}{\cos x}$

$y' = \frac{\cos x(1) - x(-\sin x)}{\cos^2 x}$

$y' = \frac{\cos x + x\sin x}{\cos^2 x}$

g. $f(\theta) = \frac{\sec\theta}{1+\sec\theta}$

$f'(\theta) = \frac{(1+\sec\theta)(\sec\theta\tan\theta) - \sec\theta(\sec\theta\tan\theta)}{(1+\sec\theta)^2}$

$= \frac{\sec\theta\tan\theta + \sec^2\theta\tan\theta - \sec^2\theta\tan}{(1+\sec\theta)^2}$

$f'(\theta) = \frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}$

h. $y = \frac{\tan x - 1}{\sec x}$

$y' = \frac{\sec x(\sec^2 x) - (\tan x - 1)(\sec x\tan x)}{\sec^2 x}$

$y' = \frac{\sec^3 x - \sec x\tan^2 x - \sec x\tan x}{\sec^2 x}$

i. $y = \sec\theta\tan\theta$

$y' = \sec\theta(\sec^2\theta) + \tan\theta(\sec\theta\tan\theta)$

$= \sec^3\theta + \sec\theta\tan^2\theta$

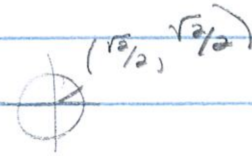
j. $f(x) = \sqrt{x}\sin x = x^{1/2}\sin x$

$f'(x) = x^{1/2}(\cos x) + \sin x\left(\frac{1}{2}x^{-1/2}\right)$

$\sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}$

24. a. $y = \tan x$ at $(\pi/4, 1)$

$y' = \sec^2 x$



$y'(\pi/4) = (\sec \pi/4)^2 = (\sqrt{2})^2 = 2$

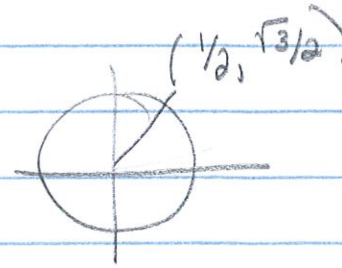
$y - 1 = 2(x - \pi/4)$

b. $y = x + \cos x$ at $(0, 1)$

$y' = 1 - \sin x$

$y'(0) = 1 - \sin 0 = 1$

$y - 1 = 1(x - 0)$



c. $y = \sec x - 2\cos x$ at $(\frac{\pi}{3}, 1)$

$y' = \sec x \tan x + 2\sin x$

$\sec \pi/3 \tan \frac{\pi}{3} + 2\sin \pi/3$

$2(\sqrt{3}) + 2(\frac{\sqrt{3}}{2})$

$2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

$y - 1 = 3\sqrt{3}(x - \pi/3)$

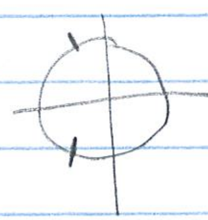
25. $f(x) = x + 2\sin x$

$f'(x) = 1 + 2\cos x$ Horizontal $f'(x) = 0$

$0 = 1 + 2\cos x$

$2\cos x = -1$

$\cos x = 1/2$



$x = \frac{2\pi}{3} + 2n\pi$

$\frac{4\pi}{3} + 2n\pi$

26. a. $y = \sin(4x)$

$y' = \cos(4x)\,4$

$y' = 4\cos(4x)$

b. $y = \sqrt{4+3x} = (4+3x)^{1/2}$

$y' = \frac{1}{2}(4+3x)^{-1/2}(3)$

$y' = \dfrac{3}{2\sqrt{4+3x}}$

c. $y = (1-x^2)^{10}$

$y' = 10(1-x^2)^9(-2x)$

$y' = -20x(1-x^2)^9$

d. $y = \tan(\sin x)$

$y' = \sec^2(\sin x)(\cos x)$

$y' = \cos x\,(\sec^2(\sin x))$

e. $y = \sqrt{\sin x} = (\sin x)^{1/2}$

$y' = \frac{1}{2}(\sin x)^{-1/2}(\cos x)$

$y' = \dfrac{-\cos x}{2\sqrt{\sin x}}$

f. $y = \sin\sqrt{x} = \sin(x^{1/2})$

$y' = \cos(x^{1/2})\cdot\left(\frac{1}{2}x^{-1/2}\right)$

$\dfrac{\cos\sqrt{x}}{2\sqrt{x}}$

g. $F(x) = (x^3+4x)^7$

$F'(x) = 7(x^3+4x)^6(3x^2+4)$

h. $F(x) = (x^2 - x + 1)^3$

$F'(x) = 3(x^2 - x + 1)^2 (2x - 1)$

i $F(x) = \sqrt[4]{1 + 2x + x^3} = (1 + 2x + x^3)^{1/4}$

$F'(x) = \frac{1}{4}(1 + 2x + x^3)^{-3/4} (2 + 3x^2)$

$F'(x) = \dfrac{(2 + 3x^2)}{4(1 + 2x + x^3)^{3/4}}$

j $f(x) = (1 + x^4)^{2/3}$

$f'(x) = \frac{2}{3}(1 + x^4)^{-1/3} (4x^3)$

$\dfrac{8x^3}{3(1 + x^4)^{1/3}}$

K. $g(t) = \dfrac{1}{(t^4 + 1)^3} = (t^4 + 1)^{-3}$

$g'(t) = -3(t^4 + 1)^{-4} (4t^3)$

$g'(t) = \dfrac{-12t^3}{(t^4 + 1)^4}$

l. $f(t) = \sqrt[3]{1 + \tan t} = (1 + \tan t)^{1/3}$

$f'(t) = \frac{1}{3}(1 + \tan t)^{-2/3} \cdot \sec^2 t$

$f'(t) = \dfrac{\sec^2 t}{3(1 + \tan t)^{2/3}}$

m. $y = \cot\left(\frac{x}{2}\right)$

$y' = -\csc^2\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)$

n. $y = 4 \sec 5x$

$y' = 4 \sec(5x) \tan(5x)\, 5$

$y' = 20 \sec(5x) \tan(5x)$

27. $y = x^3 \cos 5x$

a. $y' = x^3 \cdot (-\sin 5x)(5) + \cos 5x\,(3x^2)$

$y' = -5x^3 \sin 5x + 3x^2 \cos 5x$

b. $y = \dfrac{x}{\sqrt{7-3x}} = \dfrac{x}{(7-3x)^{1/2}}$

$y' = \dfrac{(7-3x)^{1/2}(1) - x \cdot \frac{1}{2}(7-3x)^{-1/2}}{7-3x}$

$y' = \dfrac{7-3x - \frac{x}{2}(7-3x)^{-1/2}}{7-3x}$ will simplify later in semester

c. $y = \sin\sqrt{1+x^2} = \sin(1+x^2)^{1/2}$

$-\cos(1+x^2)^{1/2} \cdot \frac{1}{2}(1+x^2)^{-1/2} \cdot (2x)$

$\dfrac{-x\cos\sqrt{1+x^2}}{\sqrt{1+x^2}}$

d. $y = x \sin \frac{1}{x} = x \sin x^{-1}$

$y' = x \cdot \cos(x^{-1}) \cdot (-1x^{-2}) + \sin(x^{-1})(1)$

$y' = \dfrac{-x\cos(\frac{1}{x})}{x^2} + \sin(1/x)$

28. $f(x) = 2\sin x + \sin^2 x = 2\sin x + (\sin x)^2$

$f'(x) = 2\cos x + 2(\sin x)^1(-\cos x)$

$\dfrac{0 = 2\cos x - 2\sin x\cos x}{2}$ horizontal line set $=0$

$0 = \cos x\,(1 - \sin x)$

$\cos x = 0 \qquad \sin x = 1$

$\boxed{x = \pi/2 + n\pi}$

29. $F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(3) = f'(g(3)) \cdot g'(3)$

$F'(3) = f'(6) \cdot 4$

$7 \cdot 4 = 28$