

Solutions to Parametric HW #4

1. $\frac{dy}{dx} = \frac{3\cos(3t)}{2e^{2t}}$

2. Length = $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 4\sin^2 t \cos^2 t} dt$

3. $\frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t}$ is undefined when $3t^2 - 2t = 0$.

So the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ has a vertical tangent when $t = 0$ and $t = \frac{2}{3}$.

4. $v(t) = \left\langle 2t, \frac{2}{2t+3} \right\rangle$, $a(t) = \left\langle 2, -\frac{4}{(2t+3)^2} \right\rangle$

5. $\left. \frac{dy}{dx} \right|_{t=1} = \left. \frac{3t^2 - 4}{6t - 1} \right|_{t=1} = -\frac{1}{2}$. When $t = 1$, $x = 1$, $y = -3$.

Tangent line equation: $y + 3 = -\frac{1}{2}(x - 1)$

6. $e^t = x - 1$ so $e^{2t} = x^2 - 2x + 1$. Then $y = 2e^{2t}$ so $y = 2x^2 - 4x + 2$.

7. Speed = $\sqrt{(-5\sin(5t))^2 + (3t^2)^2} \Big|_{t=2} = 12.304$

8. (a) Magnitude = $\sqrt{(t-2)^4 + (2t-4)^2} \Big|_{t=1} = \sqrt{5}$

(b) Distance = $\int_0^1 \sqrt{(t-2)^4 + (2t-4)^2} dt = 3.816$

(c) The particle is at rest when $v(t) = \langle (t-2)^2, 2t-4 \rangle = \langle 0, 0 \rangle$, so is at rest when $t = 2$. Position = $(4, 0)$

9. $a(5) = \langle 10.178, 6.277 \rangle$, speed = $\sqrt{(1 + \tan(t^2))^2 + (3e^{t^2})^2} \Big|_{t=5} = 28.083$

10. $3t + 2\sin t = 5$ when $t = 1.079\dots$

$v(1.079\dots) = \langle 0.119, 3.944 \rangle$

11. (a) $\left. \frac{dy}{dx} = \frac{\cos(t^2)}{2\sin(t^3)} \right|_{t=1} = 0.321$

Tangent line equation: $y - 4 = 0.321(x - 3)$

(b) Speed = $\sqrt{4\sin^2(t^3) + \cos^2(t^2)} \Big|_{t=2} = 2.084$

(c) Distance = $\int_0^1 \sqrt{4\sin^2(t^3) + \cos^2(t^2)} dt = 1.126$

(d) $x(2) = 3 + \int_1^2 2\sin(t^3) dt = 3.436$, $y(2) = 4 + \int_1^2 \cos(t^2) dt = 3.557$ so position = $(3.436, 3.557)$