Solutions to Parametric HW #3

1. $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{\frac{d}{dt}\left[e^{t^3}\right]}{\frac{d}{dt}\left[t^2-1\right]}=\dfrac{3t^2e^{t^3}}{2t}=\dfrac{3te^{t^3}}{2}$.

2. $v(t)=\left\langle \dfrac{dx}{dt}, \dfrac{dy}{dt}\right\rangle=\left\langle \dfrac{2t+5}{t^2+5t}, 6t\right\rangle$ so $v(2)=\left\langle \dfrac{9}{14}, 12\right\rangle$.

3. $v(t)=\left\langle \dfrac{dx}{dt}, \dfrac{dy}{dt}\right\rangle=\left\langle 5t^4, 12t^3-6t^2\right\rangle$.

   $a(t)=\left\langle \dfrac{d^2x}{dt^2}, \dfrac{d^2y}{dt^2}\right\rangle=\left\langle 20t^3, 36t^2-12t\right\rangle$, so $\mathrm{a}(1)=\langle 20, 24\rangle$.

4. $v(t)=\left\langle \dfrac{dx}{dt}, \dfrac{dy}{dt}\right\rangle=\left\langle 3\cos\left(3t-\dfrac{\pi}{2}\right), 6t\right\rangle$ so $v\left(\dfrac{\pi}{2}\right)=\langle 3\cos\pi, 3\pi\rangle=\langle -3, 3\pi\rangle$.

5. $x(t)=\int(t+1)\,dt=\dfrac{t^2}{2}+t+C$.

   $x(0)=1=C$ so $x(t)=\dfrac{t^2}{2}+t+1$.

   Since $x(1)=\dfrac{5}{2}$ and $y(1)=\ln\left(\dfrac{5}{2}\right)$, Position $=\left(\dfrac{5}{2}, \ln\left(\dfrac{5}{2}\right)\right)$.

   Or, since $x(1)=1+\int_0^1(t+1)\,dt=\dfrac{5}{2}$ and $y(1)=\ln\left(\dfrac{5}{2}\right)$, Position $=\left(\dfrac{5}{2}, \ln\left(\dfrac{5}{2}\right)\right)$.

6. $x(t)=\int(1+t)\,dt=t+\dfrac{t^2}{2}+C$. $x(0)=5$ so $C=5$ and $x(t)=t+\dfrac{t^2}{2}+5$.

   $y(t)=\int t^3\,dt=\dfrac{t^4}{4}+D$. $y(0)=0$ so $D=0$ and $y(t)=\dfrac{t^4}{4}$.

   Position vector $=\left\langle t+\dfrac{t^2}{2}+5, \dfrac{t^4}{4}\right\rangle$. At $t=2$, Position $=(9, 4)$.

   Or, since $x(2)=5+\int_0^2(1+t)\,dt=9$ and $y(2)=0+\int_0^2 t^3\,dt=4$,

   Position $=(9, 4)$.

7. When $x = 2, y = 5$.

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$(2)(3) + 5\frac{dx}{dt} = 0$ so $\frac{dx}{dt} = -\frac{6}{5}$

Or find that $\frac{dy}{dx} = -\frac{10}{x^2}$. Then substituting into $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ gives $\frac{-10}{4} = \frac{3}{\frac{dx}{dt}}$ so that $\frac{dx}{dt} = -\frac{6}{5}$.

8. $x'(t) = 3t^2 - 3t - 18 = 3(t-3)(t+2) = 0$ when $t = 3$ and $t = -2$.

$y'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$ when $t = 3$ and $t = 1$.

The particle is at rest when $v(t) = \langle 0, 0\rangle$ so at rest when $t = 3$.

9. $t^3 = 8$ $\quad$ $t^2 - 5t + 2 = -4$

$t = 2$ $\quad$ $t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t = 3,\ t = 2$

At $(8, -4)$ when $t = 2$

$$\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{2t-5}{3t^2}\right|_{t=2} = -\frac{1}{12}$$

Tangent line equation: $y + 4 = -\frac{1}{12}(x-8)$

10. $5t + 3\sin t = 25$ when $t = 5.445755...$

$$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle 5 + 3\cos t, -1 + \cos t + (8-t)\sin t\rangle$$

$$v(5.445755...) = \langle 7.008, -2.228\rangle$$

11. **(a)** Magnitude when $t = 5$ is $\left.\sqrt{(2t)^2 + (2t^2)^2}\right|_{t=5} = \sqrt{2600}$ or $10\sqrt{26}$

(b)

Distance $= \int_0^5 \sqrt{(2t)^2 + (2t^2)^2}\,dt = \int_0^5 2t\sqrt{1+t^2}\,dt = \left.\frac{2}{3}(1+t^2)^{\frac{3}{2}}\right|_0^5 = \frac{2}{3}\left(26^{\frac{3}{2}} - 1\right)$

(c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t = \sqrt{x+3}$

12. **(a)** $x(t)=\int \frac{1}{t+1}dt=\ln(t+1)+C$. When $t=1, x=\ln 2$ so $C=0$.

$x(t)=\ln(t+1)$

$y(t)=\int 2t\,dt=t^2+D$. When $t=1, y=0$ so $D=-1$.

$y(t)=t^2-1$

$(x,y)=\left(\ln(t+1), t^2-1\right)$

(b) $t+1=e^x$ so $t=e^x-1$ and $y=\left(e^x-1\right)^2-1=e^{2x}-2e^x$.

(c) Average rate of change $=\frac{y(b)-y(a)}{x(b)-x(a)}=\frac{y(4)-y(0)}{x(4)-x(0)}=\frac{15-(-1)}{\ln 5-\ln 1}=\frac{16}{\ln 5}$

(d) Instantaneous rate of change $=\left.\frac{dy}{dx}\right|_{t=1}=\left.\frac{2t}{\frac{1}{t+1}}\right|_{t=1}=4$

13. **(a)** $\frac{dy}{dx}=\frac{2\cos t}{3\sin t}=\frac{2}{3}\cot t$

(b) $y-\left(3+\sqrt{2}\right)=\frac{2}{3}\left(x-\left(2-\frac{3\sqrt{2}}{2}\right)\right)$

(c) $x=0$ when $t=-0.84106867\ldots, 0.84106867\ldots$

length $=\int_{-0.841\ldots}^{0.841\ldots}\sqrt{(3\sin t)^2+(2\cos t)^2}\,dt=3.756$ or 3.757