Solutions to Parametric HW #2

1. Since $x'(t) = 6t^2 - 42t + 72 = 6(t^2 - 7t + 12) = 6(t - 3)(t - 4) = 0$ when $t = 3$ and when $t = 4$, the answer is E.

2. Note that $a(t) = 3t^2 - 6t + 12$, so that $a'(t) = 6t - 6 = 0$ when $t = 1$. Computing the acceleration at the critical number and at the endpoints of the interval, we have $a(0) = 12$, $a(1) = 9$, and $a(3) = 21$. The maximum acceleration is 21, so the answer is D.

3. Note that $v(t) = 6t^2 - 48t + 90 = 6(t - 3)(t - 5)$ and $a(t) = 12t - 48 = 12(t - 4)$. The speed is increasing on $3 < t < 4$, where the velocity and the acceleration are both negative, and also for $t > 5$, where the velocity and the acceleration are both positive, so the answer is E.

4. Since $\frac{d}{dt}\left[3+4.1\cos(0.9t)\right]_{t=4} = 1.633$, the answer is C.

5. Since $v(2) = 2 + \int_1^2 \ln(1+2^t)\,dt = 3.346$, the answer is E.

6. First find $\frac{d}{dt}[\sin t] = \cos t$ and $\frac{d}{dt}\left[e^{-2t}\right] = -2e^{-2t}$. Then graph $y_1 = \cos x$ and $y_2 = -2e^{-2x}$ in function mode with an x-window of [0, 10] and a y- window of [−1 , 1]. The two graphs intersect at three points, so the answer is D.

7. Distance = $\int_0^2 |v(t)|\,dt = \int_0^2 \left|3e^{(-t/2)}\sin(2t)\right|dt = 2.261$, so the answer is D.

8. (a) $a(2) = v'(2) = -0.132$ or -0.133.

   (b) $v(2) = -0.436$. Since $a(2) < 0$, and $v(2) < 0$, the speed is increasing.

   (c) Note that $v(t) = 0$ when $\tan^{-1}(e^t) = 1$. The only critical number for y is $t = \ln(\tan 1) = 0.443$. Since $v(t) > 0$ for $0 \le t < \ln(\tan 1)$ and $v(t) < 0$ for $t > \ln(\tan 1)$, $y(t)$ has an absolute maximum at $t = 0.443$.

   (d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361.

Since $v(2) < 0$ and $y(2) < 0$, the particle is moving away from the origin.

9. (a) Average acceleration of rocket A is

$$\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} = \frac{11}{20}\text{ ft / sec}^2.$$

   (b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is

$$20\left[v(20)+v(40)+v(60)\right] = 20(22+35+44) = 2020\text{ ft}.$$

   (c) Let $v_B(t)$ be the velocity of rocket B at time t. Then $v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$. Since $2 = v_B(0) = 6 + C$, then $C = -4$ and $v_B(t) = 6\sqrt{t+1} - 4$. Hence, $v_B(80) = 50 > 49 = v(80)$ and Rocket B is traveling faster at time $t = 80$ seconds.