Summer Assignment Advanced Placement Calculus BC
Bear Creek High School
Instructor: Ms. Killeen

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Sign up for Remind: Directions on website
Text the message @bccalcsum to the number 81010
(receive and send messages—more timely method of communication)

Congratulations and thank you for signing up for Advanced Placement (AP) Calculus BC.

This summer assignment is a very important first step in successfully completing AP Calculus. It is important that you are prepared from the very beginning of the course to ensure that you will do well and achieve your ultimate goal, a 3 or higher on the AP Exam. You do not want the fact that you do not remember important skills from previous math classes to prevent you from achieving this goal.

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in Calculus. There are lots of problems, but the majority of them can be completed in less than three steps. You should work all parts of every question. The assignment is split in two parts. Part I covers skills in Algebra 2 and Pre-Calculus and may be written in the packet. Part II covers limits and basic derivatives and MUST be completed on loose-leaf paper. The work will be graded according to a rubric that will take into account the following items: the completion of each problem, the accurate completion of selected problems, and the organization/neatness of the entire assignment. Multiple assessments will be given in the first two weeks of school over this assignment. You may work together or use reference materials to assist you and refresh your memory (old notes, textbooks, online resources such as Khan Academy or mathispower4u.com, etc.)

It is a mistake to decide to do this now. Let it go until the beginning of August. I need these techniques to be fresh in your mind in the fall. However be careful to not wait to do them at the very last minute. These take time. I am predicting about 11 hours total. I don't even like to do 11 hours of math in one sitting, so schedule your time and plan accordingly.

This assignment will be collected on Friday, August 17, 2018. Be sure to show all appropriate work to support your answers.

This is a college course, so you will be required to purchase your textbook:
You must have your book by August 27, but ideally the first day of school.
Information about the specific book is on the back of this letter.

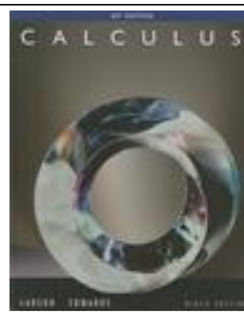
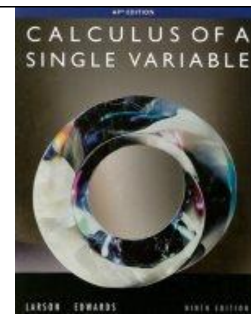
Thank you. I am looking forward to a fantastic 2018-2019 school year.
Have a great summer!

BC Calculus

2 options:

1: Calculus Ninth Edition: This option is for incoming sophomores and juniors. This book contains the content for BC Calculus as well as Multivariate. If you are planning on taking Multivariate at Bear Creek High School, pick this option, because this book will be used the following year.

2: Calculus of a Single Variable Ninth Edition: This option is primarily for incoming seniors. This book contains all the content for BC Calculus.

Option 1 **(AP EDITION)** **CALCULUS** **NINTH EDITION** **Larson, Edwards** **Brooks/Cole Cengage Learning** **Copyright 2010**  **ISBN-13:978-0-547-21289-0**	**Option 2** **CALCULUS** **OF A SINGLE VARIABLE** **NINTH EDITION** **AP Edition** **Larson, Edwards** **Brooks/Cole** **Copyright 2010**  **ISBN-13: 978-0-547-1290-6** **ISBN-10: 0-547-21290-9**

This book has been published for a while. You should be able to find it for a reasonable price by either purchasing a used book or finding an online version.

AP Calculus BC
2018-2019
Ms. Killeen

Summer Assignment

Bear Creek High School

AP Calculus Summer Review Packet

Name:______________________________

READ THE FOLLOWING DIRECTIONS CAREFULLY!

1. Before answering any questions read through the given notes and examples for each topic.

2. This packet is to be handed in on Friday, August 18.

3. All work must be shown in the packet (part I) and on separate paper (part II).

4. Completion of this packet is worth 100 points.

5. There will be a 10 point dock for each day the packet is turned in late.

6. ALL work must be organized and neatly written.

You may seek assistance from any source as long as the end result is the answer "yes" to the following questions:

- Do I understand how to work the problem?
- Can I rework the problem myself?
- Can I explain it to another person?
- Can I work another problem similar to it?
- Do I know the concepts required to work this problem?
- Could I extend my knowledge of this problem to apply it to another application?
- Could I use my knowledge of this problem to complete a problem on a test that requires the use of concepts developed in this problem?

PART I **FUNCTIONS** (show work on packet sheet)

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x)).*

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

1. $f(2) =$ ____________ 2. $g(-3) =$ ______________ 3. $f(t+1) =$ __________

4. $f\left[g(-2)\right] =$ __________ 5. $g\left[f(m+2)\right] =$ ___________ 6. $\left[f(x)\right]^2 - 2g(x) =$ ______

Let $f(x) = \sin(2x)$ **Find each exactly.**

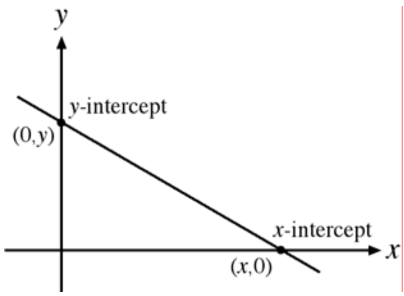
7. $f\left(\frac{\pi}{4}\right) =$ ___________ 8. $f\left(\frac{2\pi}{3}\right) =$ ______________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

9. $h\left[f(-2)\right] =$ _______ 10. $f\left[g(x-1)\right] =$ _______ 11. $g\left[h(x^3)\right] =$ _______

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

$\underline{x - \text{int. } (Let\ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1 \ or \ x = 3$

$x - \text{intercepts } (-1,0) \ and \ (3,0)$

$\underline{y - \text{int. } (Let\ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept } (0,-3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

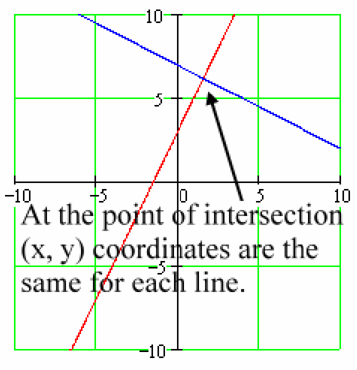
13. $y = x^2 + x - 2$

14. $y = x\sqrt{16 - x^2}$

15. $y^2 = x^3 - 4x$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2nd Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{array}{l} x^2 - y = 3 \\ x - y = 1 \end{array}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{array}{l} x + y = 8 \\ 4x - y = 7 \end{array}$

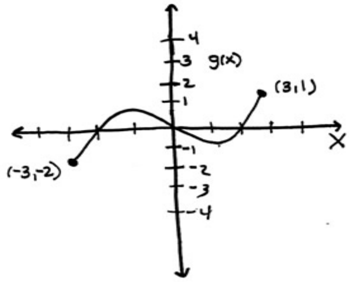
17. $\begin{array}{l} x^2 + y = 6 \\ x + y = 4 \end{array}$

18. $\begin{array}{l} x = 3 - y^2 \\ y = x - 1 \end{array}$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).

The domain is the set of inputs of the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x)=x^2-5$

20. $f(x)=-\sqrt{x+3}$

21. $f(x)=3\sin x$

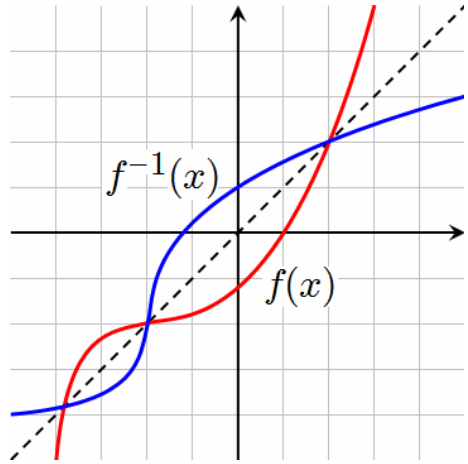
22. $f(x)=\dfrac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x)=2x+1$

24. $f(x)=\dfrac{x^2}{3}$

25. $g(x)=\dfrac{5}{x-2}$

26. $y=\sqrt{4-x}+1$

27. If the graph of $f(x)$ has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

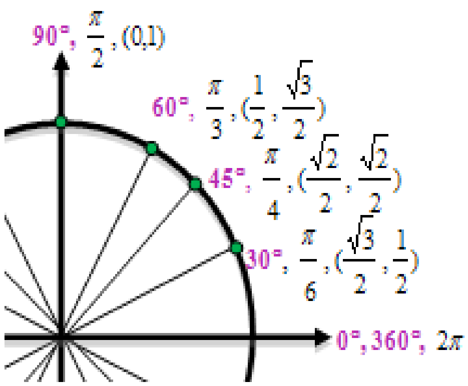
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

34. Find the equation of a line passing through the points (-3, 6) and (1, 2).

35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\quad \cos\frac{\pi}{2} = 0$ $\quad \tan\frac{\pi}{2} = und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. *a.*) $\sin\pi$ *b.*) $\cos\frac{3\pi}{2}$ *c.*) $\sin\left(-\frac{\pi}{2}\right)$ *d.*) $\sin\left(\frac{5\pi}{4}\right)$

e.) $\cos\frac{\pi}{4}$ *f.*) $\cos(-\pi)$ g) $\cos\frac{\pi}{3}$ h) $\sin\frac{5\pi}{6}$

i) $\cos\frac{2\pi}{3}$ j) $\tan\frac{\pi}{4}$ k) $\tan\pi$ l) $\tan\frac{\pi}{3}$

m) $\cos\frac{4\pi}{3}$ n) $\sin\frac{11\pi}{6}$ o) $\tan\frac{7\pi}{4}$ p) $\sin\left(-\frac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \le x < 2\pi$ **.**

37. $\sin x = -\frac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$

**Recall $\sin^2 x = (\sin x)^2$

**Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x-c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x+c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

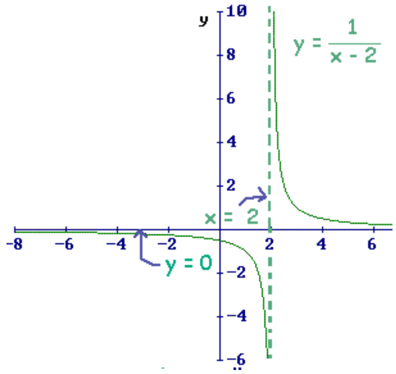
a) $f(x) - 3$ b) $f(x-3)$ c) $2f(x)$ d) $f(x-2)+1$ e) $-f(x)$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$

Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \frac{1}{x^2}$

45. $f(x) = \frac{x^2}{x^2 - 4}$

46. $f(x) = \frac{2+x}{x^2(1-x)}$

47. $f(x) = \frac{4-x}{x^2 - 16}$

48. $f(x) = \frac{x-1}{x^2 + x - 2}$

49. $f(x) = \frac{5x+20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

51. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52. $f(x) = \frac{4x^2}{3x^2 - 7}$

53. $f(x) = \frac{(2x-5)^2}{x^2 - x}$

54. $f(x) = \frac{-3x + 1}{\sqrt{x^2 + x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$ Get a common base

$2^{2x+2} = 2^{-3x+2}$ Simplify

$2x + 2 = -3x + 2$ Set exponents equal

$x = 0$ Solve for x

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^{x} = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^{x} = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828… or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^{?} = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

60. $\log_2 \frac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln\sqrt{e}$

65. $\ln\frac{1}{e}$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\log_b x^y = y \log_b x$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9 \ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 \left(\sqrt{2}\right)^5$

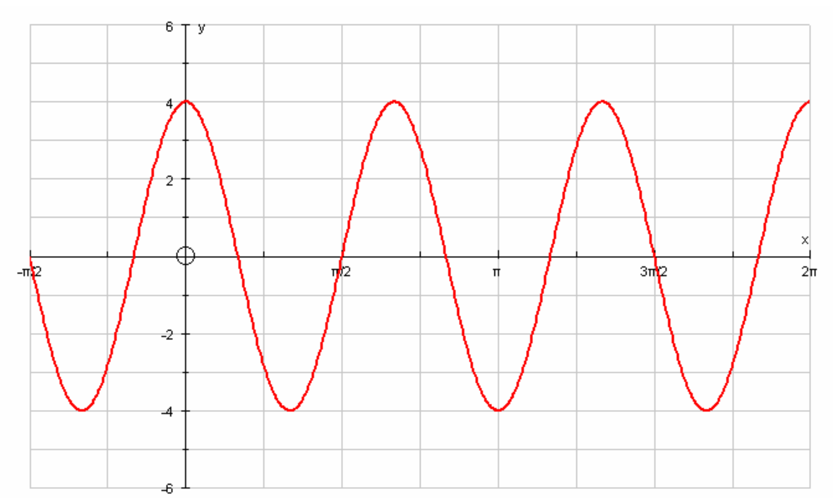
EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

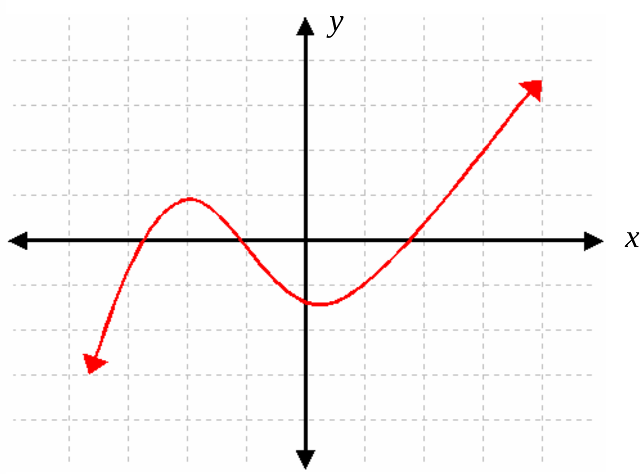
Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ______________________



78. ______________________



79. ______________________
$f(x) = 2x^4 - 5x^2$

80. ______________________
$g(x) = x^5 - 3x^3 + x$

81. ______________________
$h(x) = 2x^2 - 5x + 3$

82. ______________________
$j(x) = 2\cos x$

83. ______________________
$k(x) = \sin x + 4$

84. ______________________
$l(x) = \cos x - 3$

23. Differentiate (trig, product and quotient)

a. $y = x - 3\sin x$

b. $y = x\sin x$

c. $y = \sin x + 10\tan x$

d. $y = 2\csc x + 5\cos x$

e. $g(t) = 4\sec t + \tan t$

f. $y = \dfrac{x}{\cos x}$

g. $f(\theta) = \dfrac{\sec\theta}{1+\sec\theta}$

h. $y = \dfrac{\tan x - 1}{\sec x}$

i. $y = \sec\theta\tan\theta$

j. $f(x) = \sqrt{x}\sin x$

24. Find an equation of the tangent line to the curve at the given point.

a. $y = \tan x$ at $\left(\dfrac{\pi}{4}, 1\right)$

b. $y = x + \cos x$ at $(0,1)$

c. $y = \sec x - 2\cos x$ at $\left(\dfrac{\pi}{3}, 1\right)$

25. Find where the curve has a horizontal tangent: $f(x) = x + 2\sin x$.

26. Differentiate. (Chain)

a. $y = \sin 4x$

b. $y = \sqrt{4+3x}$

c. $y = (1-x^2)^{10}$

d. $y = \tan(\sin x)$

e. $y = \sqrt{\sin x}$

f. $y = \sin\sqrt{x}$

g. $F(x) = (x^3+4x)^7$

h. $F(x) = (x^2 - x + 1)^3$

i. $F(x) = \sqrt[4]{1+2x+x^3}$

j. $f(x) = (1+x^4)^{2/3}$

k. $g(t) = \dfrac{1}{(t^4+1)^3}$

l. $f(t) = \sqrt[3]{1+\tan t}$

m. $y = \cot\left(\dfrac{x}{2}\right)$

n. $y = 4\sec 5x$

27. Differentiate

a. $y = x^3\cos 5x$

b. $y = \dfrac{x}{\sqrt{7-3x}}$

c. $y = \sin\sqrt{1+x^2}$

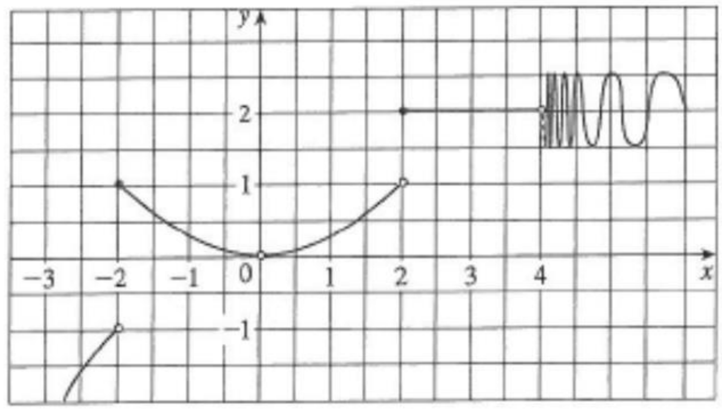
d. $x\sin\dfrac{1}{x}$

28. Find all points on the graph of $f(x) = 2\sin x + \sin^2 x$ **at which the tangent line is horizontal.**

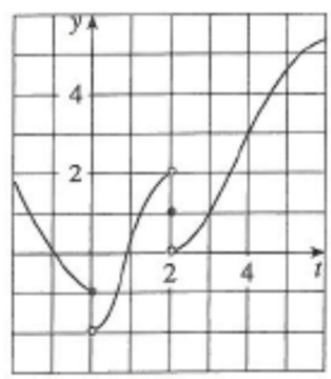
29. Suppose $F(x) = f(g(x))$ **and** $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$. **Find** $F'(3)$.

show work on separate paper

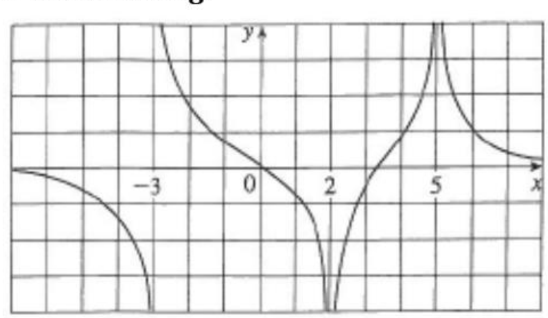
1. **For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.**
   a. $\lim_{x\to -2^-} g(x)$
   b. $\lim_{x\to -2^+} g(x)$
   c. $\lim_{x\to -2} g(x)$
   d. $g(-2)$
   e. $\lim_{x\to 2^-} g(x)$
   f. $\lim_{x\to 2^+} g(x)$
   g. $\lim_{x\to 2} g(x)$
   h. $g(2)$
   i. $\lim_{x\to 4^+} g(x)$
   j. $\lim_{x\to 4^-} g(x)$
   k. $g(0)$



2. **For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.**
   a. $\lim_{t\to 0^-} g(t)$
   b. $\lim_{t\to 0^+} g(t)$
   c. $\lim_{t\to 0} g(t)$
   d. $\lim_{t\to 2^-} g(t)$
   e. $\lim_{t\to 2^+} g(t)$
   f. $\lim_{t\to 2} g(t)$
   g. $g(2)$
   h. $\lim_{t\to 4} g(t)$



3. **For the function R whose graph is shown, state the following.**
   a. $\lim_{x\to 2} R(x)$
   b. $\lim_{x\to 5} R(x)$
   c. $\lim_{x\to -3^-} R(x)$
   d. $\lim_{x\to -3^+} R(x)$
   e. The equations of the vertical asymptotes



4. **Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists:**

$$f(x)=\begin{cases} 2-x, & \text{for } x<-1 \\ x, & \text{for } -1\le x<1 \\ (x-1)^2, & \text{for } x\ge 1 \end{cases}$$

5. **Determine the infinite limit**

a. $\lim_{x\to 5^+} \frac{6}{x-5}$

b. $\lim_{x\to 5^-} \frac{6}{x-5}$

c. $\lim_{x\to 1} \frac{2-x}{(x-1)^2}$

d. $\lim_{x\to 0} \frac{x-1}{x^2(x+2)}$

e. $\lim_{x\to -2^+} \frac{x-1}{x^2(x+2)}$

6. **Given that $\lim_{x\to a} f(x)=-3$, $\lim_{x\to a} g(x)=0$, and $\lim_{x\to a} h(x)=8$, find the limits that exists. If a limit does not exist, explain why.**

a. $\lim_{x\to a}\left[f(x)+h(x)\right]$

b. $\lim_{x\to a}\left[f(x)\right]^2$

c. $\lim_{x\to a}\sqrt[3]{h(x)}$

d. $\lim_{x\to a}\frac{1}{f(x)}$

e. $\lim_{x\to a}\frac{f(x)}{h(x)}$

f. $\lim_{x\to a}\frac{g(x)}{f(x)}$

g. $\lim_{x\to a}\frac{f(x)}{g(x)}$

h. $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)}$

7. **Evaluate each limit**

a. $\lim_{x\to -2}\left(3x^4+2x^2-x+1\right)$

b. $\lim_{x\to 2}\frac{2x^2+1}{x^2+6x-4}$

c. $\lim_{t\to -1}\left(t^2+1\right)^3(t+3)^5$

d. $\lim_{x\to 1}\left(\frac{1+3x}{1+4x^2+3x^4}\right)^3$

e. $\lim_{x\to 4^-}\sqrt{16-x^2}$

8. **Evaluate each limit if it exists (use algebra!)**

a. $\lim_{t\to -3}\frac{t^2-9}{2t^2+7t+3}$

b. $\lim_{h\to 0}\frac{(4+h)^2-16}{h}$

c. $\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}$

d. $\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

9. **Let** $f(x)=\begin{cases} 4-x^2, & x\le 2 \\ x-2, & x>2 \end{cases}$.

a. Find $\lim_{x\to 2^-} f(x)$

b. Find $\lim_{x\to 2^+} f(x)$

c. Does $\lim_{x\to 2} f(x)$ exist?

10. **Use the definition of continuity and the properties of limits to show that the function $f(x)=x^2+\sqrt{7-x}$ is continuous at $x=4$.**

11. **Find the x-values at which f is discontinuous.**

$$f(x)=\begin{cases} 1+x^2, & x\le 0 \\ 2-x, & 0<x\le 2 \\ (x-2)^2, & x>2 \end{cases}$$

12. **For what value of the constant c is $f(x)$ continuous on $(-\infty,\infty)$ if** $f(x)=\begin{cases} cx+1, & x\le 3 \\ cx^2-1, & x>3 \end{cases}$?

13. **Use the limit definition of the derivative** $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ **to find** $f'(a)$.

a. $f(x)=3-2x+4x^2$, at $a=1$

b. $f(x)=\frac{1}{\sqrt{x+2}}$, at $a=7$

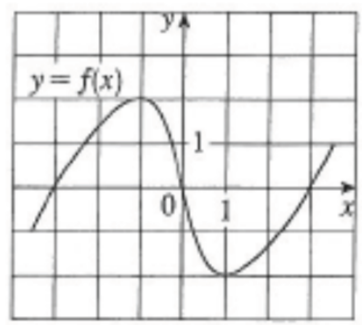
<u>FROM THIS POINT ON, YOU CAN USE RULES FOR DERIVATIVES... NOT A LIMIT DEFINITION</u>
<u>Leave no negative exponents in your final answers.</u>

14. If $g(x)=1-x^3$, find $g'(0)$ and use it to find an equation of the tangent line to $g(x)$ at $(0,1)$

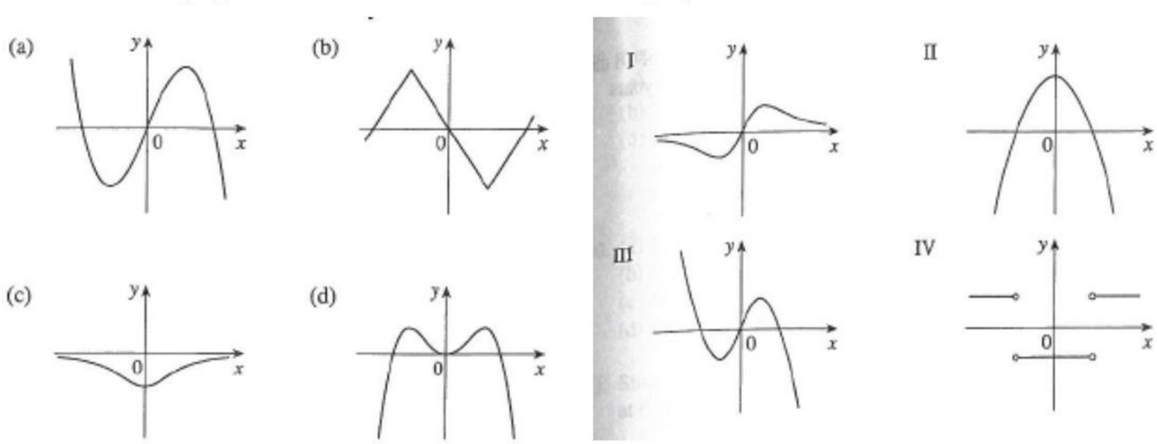
15. If $f(x)=3x^2-5x$, find $f'(2)$ and use it to find an equation of the tangent line to $f(x)$ at $(2,2)$

16. **Use the graph to estimate the slope at each point (sketch a tangent line at each point and approximate its slope). Then use the values to sketch** $f'(x)$

a. $f'(-3)$
b. $f'(-2)$
c. $f'(-1)$
d. $f'(0)$
e. $f'(1)$
f. $f'(2)$
g. $f'(3)$

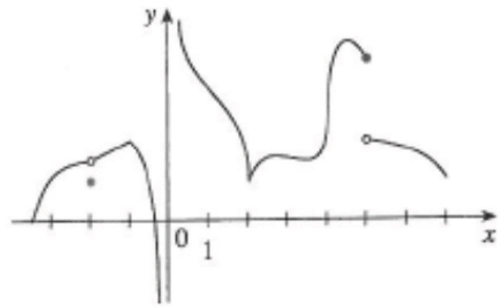


17. **Match the graph of each function in a-d with the graph of its derivative.**



18. **The graph of g is given.**

   a. At what numbers is g discontinuous? Why?
   b. At what numbers is g not differentiable? Why?



19. **Differentiate each function. (Power Rule)**

   a. $f(x)=186.5$
   b. $f(x)=\sqrt{30}$
   c. $f(x)=5x-1$
   d. $F(x)=-4x^{10}$
   e. $g(x)=5x^8-2x^5+6$
   f. $f(t)=\frac{1}{4}(t^4+8)$
   g. $f(t)=\frac{1}{2}t^6-3t^4+t$
   h. $V(r)=\frac{4}{3}\pi r^3$
   i. $R(t)=5t^{-3/5}$
   j. $Y(t)=6t^{-9}$
   k. $R(x)=\frac{\sqrt{10}}{x^7}$
   l. $f(t)=\sqrt{t}-\frac{1}{\sqrt{t}}$
   m. $y=x^{-2/5}$
   n. $y=\sqrt[3]{x}$
   o. $y=4\pi r^2$
   p. $g(u)=\sqrt{2}u+\sqrt{3u}$
   q. $y=x+\frac{1}{x}$

20. **Differentiate (power, product and quotient)**

   a. $y=\sqrt{x}(x-1)$
   b. $g(x)=\frac{3x-1}{2x+1}$
   c. $f(t)=\frac{2t}{4+t^2}$
   d. $y=\frac{\sqrt{x}-1}{\sqrt{x}+1}$
   e. $y=ax^2+bx+c$
   f. $y=A+\frac{B}{x}+\frac{C}{x^2}$

21. **Find the equation of the TANGENT line and PERPENDICULAR line for each function at the point. You should have two equations per problem.**

   a. $y=\frac{2x}{x+1}$ at $(1,1)$
   b. $y=\frac{\sqrt{x}}{x+1}$ at $(4,0.4)$
   c. $y=x+\sqrt{x}$ at $(1,2)$
   d. $y=(1+2x)^2$ at $(1,9)$

22. **Let $g(x)$ be a differentiable function (so $g'(x)$ exists). Find an expression for the derivative of each function (use box rule):**

   a. $y=xg(x)$
   b. $y=\frac{x}{g(x)}$
   c. $y=\frac{g(x)}{x}$

23. Differentiate (trig, product and quotient)

a. $y = x - 3\sin x$

b. $y = x\sin x$

c. $y = \sin x + 10\tan x$

d. $y = 2\csc x + 5\cos x$

e. $g(t) = 4\sec t + \tan t$

f. $y = \dfrac{x}{\cos x}$

g. $f(\theta) = \dfrac{\sec\theta}{1+\sec\theta}$

h. $y = \dfrac{\tan x - 1}{\sec x}$

i. $y = \sec\theta\tan\theta$

j. $f(x) = \sqrt{x}\sin x$

24. Find an equation of the tangent line to the curve at the given point.

a. $y = \tan x$ at $\left(\dfrac{\pi}{4}, 1\right)$

b. $y = x + \cos x$ at $(0,1)$

c. $y = \sec x - 2\cos x$ at $\left(\dfrac{\pi}{3}, 1\right)$

25. Find where the curve has a horizontal tangent: $f(x) = x + 2\sin x$.

26. Differentiate. (Chain)

a. $y = \sin 4x$

b. $y = \sqrt{4+3x}$

c. $y = (1-x^2)^{10}$

d. $y = \tan(\sin x)$

e. $y = \sqrt{\sin x}$

f. $y = \sin\sqrt{x}$

g. $F(x) = (x^3+4x)^7$

h. $F(x) = (x^2 - x + 1)^3$

i. $F(x) = \sqrt[4]{1+2x+x^3}$

j. $f(x) = (1+x^4)^{2/3}$

k. $g(t) = \dfrac{1}{(t^4+1)^3}$

l. $f(t) = \sqrt[3]{1+\tan t}$

m. $y = \cot\left(\dfrac{x}{2}\right)$

n. $y = 4\sec 5x$

27. Differentiate

a. $y = x^3\cos 5x$

b. $y = \dfrac{x}{\sqrt{7-3x}}$

c. $y = \sin\sqrt{1+x^2}$

d. $x\sin\dfrac{1}{x}$

28. Find all points on the graph of $f(x) = 2\sin x + \sin^2 x$ **at which the tangent line is horizontal.**

29. Suppose $F(x) = f(g(x))$ **and** $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$ **. Find** $F'(3)$**.**