

BC Fun - Chpt. 8

1. $\int \frac{4x^2}{x^2+9} dx$

$$\begin{array}{r} 4 - \frac{36}{x^2+9} \\ x^2+9 \overline{) 4x^2} \\ \underline{-4x^2+36} \\ -36 \end{array}$$

$\int 4 - \frac{36}{x^2+9} dx$

$u^2 = x^2 \quad a^2 = 9$
 $u = x \quad a = 3$
 $du = dx$

$4x - 12 \arctan \frac{x}{3} + C$

$\frac{1}{a} \arctan \frac{u}{a}$

2. Look in your NOTES

3. $\int_0^{\infty} \frac{1}{e^x + e^{-x}} dx$ $\frac{e^{-x}}{e^{-x}}$

OR $\int_0^{\infty} \frac{1}{e^x + \frac{1}{e^x}} dx$

$\int_0^{\infty} \frac{e^{-x}}{e^0 + e^{-2x}} dx$

$\lim_{b \rightarrow \infty} \int_0^b \frac{1}{\frac{e^{2x} + 1}{e^x}} dx$

$\lim_{b \rightarrow \infty} \int_0^b \frac{e^{-x}}{1 + e^{-2x}} dx$

$u^2 = e^{-2x} \quad a^2 = 1$
 $u = e^{-x} \quad a = 1$
 $du = -e^{-x} dx$

$\lim_{b \rightarrow \infty} \int_0^b \frac{e^x}{e^{2x} + 1} dx$

$-\left[\lim_{b \rightarrow \infty} \arctan e^{-x} \right]_0^b$
 $F(b) - F(0)$

$u^2 = e^{2x}$
 $u = e^x$
 $du = e^x dx$

$-\left[\lim_{b \rightarrow \infty} \arctan e^{-b} - \arctan e^0 \right]$
 $-\left[\lim_{b \rightarrow \infty} \arctan \frac{1}{e^b} - \arctan 1 \right]$

same idea

$-\left[0 - \pi/4 \right]$

$\pi/4$



4. $\int x^4 \ln x \, dx$ $\int \frac{\ln x}{x^4}$

$u = \ln x$ $v = \frac{x^5}{5}$

$du = \frac{1}{x} dx$ $dv = x^4 dx$

$uv - \int v du$

$\ln x \left(\frac{x^5}{5} \right) - \frac{1}{5} \int x^5 \cdot \frac{1}{x} dx$

$\frac{x^5 \ln x}{5} - \frac{1}{5} \int x^4 dx$

$\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$

5. $\int \frac{21}{x^2 - x - 12} dx \rightarrow \frac{21}{(x-4)(x+3)} = \frac{A}{x+3} + \frac{B}{x-4}$

$\int \frac{-3}{x+3} + \frac{3}{x-4} dx$

$21 = A(x-4) + B(x+3)$

Let $x=4$

$x=-3$

$21 = 7B$

$21 = -7A$

$B=3$

$A=-3$

$-3 \ln|x+3| + 3 \ln|x-4| + C$

$\ln \left| \frac{(x-4)^3}{x+3} \right| + C$

6. $\int x^2 \sin(x) dx$

$u = x^2$

$v = -\cos x$

$uv - \int v du$

$du = 2x dx$

$dv = \sin x$

$-x^2 \cos x + 2 \int x \cos x dx$

$uv - \int v du$

$u = x$

$v = \sin x$

$du = dx$

$dv = \cos x$

$-x^2 \cos x + 2 \left[x \sin x - \int \sin x dx \right]$

$-x^2 \cos x + 2x \sin x + 2 \cos x + C$

$$\lim_{a \rightarrow 0^+} \frac{F(a) - F(a)}{a - a} = \frac{0}{0}$$

$$\lim_{a \rightarrow 0^+} \int_a^a x^{-1/2} dx$$

9. $\int_1^0 \frac{1}{\sqrt{x}} dx$

2 converge

$$\lim_{x \rightarrow 0} \frac{e^x + 1}{x} = \frac{0}{0}$$

0 converge

7. $\lim_{x \rightarrow \infty} x e^{-x^3}$

$$\lim_{x \rightarrow \infty} \frac{e^{x^3}}{x} = \frac{\infty}{\infty}$$

$$10. \int_0^6 \frac{4}{\sqrt{6-x}} dx$$

$$\lim_{b \rightarrow 6^-} 4 \int_0^b (6-x)^{-1/2} dx$$

$$u = 6-x$$

$$du = -dx$$

$$-du = dx$$

$$\lim_{b \rightarrow 6^-} -4 \int_0^b u^{-1/2} dx$$

$$-8 u^{1/2} \Big|_a^b$$

$$\lim_{b \rightarrow 6^-} -8 \left[(6-x)^{1/2} \right]_0^b$$

$$-8 [F(b) - F(0)]$$

$$-8 [0 - 6^{1/2}]$$

$$\boxed{8\sqrt{6}}$$

$$11. \int_0^4 \frac{dx}{x^3}$$

$$\lim_{a \rightarrow 0^+} \int_a^4 x^{-3} dx$$

$$\lim_{a \rightarrow 0^+} \left[\frac{x^{-2}}{-2} \right]_a^4$$

$$F(4) - F(a)$$

$$\lim_{a \rightarrow 0^+} -\frac{1}{2} \left[\frac{1}{16} - \frac{1}{a^2} \right]$$

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Diverges

$$12. \int \frac{x}{\sqrt{9-x^4}} dx$$

$$a^2 = 9 \quad u^2 = x^4$$

$$a = 3 \quad u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$\arcsin \frac{u}{a} + c$$

$$\boxed{\frac{1}{2} \arcsin \frac{x^2}{3} + c}$$