



The Integral as an Accumulator Solutions

Multiple Choice

1. D $f(1) = 5 + \int_0^1 f'(t) dt = 5 + 3 = 8.$
2. A $T(5) = T(0) + \int_0^5 -110e^{-0.4t} dt = 350 - 237.78 \approx 112$
3. E $v(2) = v(1) + \int_1^2 a(t) dt = 2 + \int_1^2 \ln(1 + 2^t) dt = 2 + 1.346 = 3.346$
4. A $T = \int_7^{14} \frac{100e^{-0.1t}}{2 - e^{-3t}} dt = 124.499 \approx 125 \text{ tons}$
5. C $\cos(2 - t^2) = 0 \Rightarrow t = 0.6551; \quad x(t) = x(0) + \int_0^{0.6551} \cos(2 - t^2) dt = 3 + (-0.1835) = 2.816$
6. B I is false since $\int_0^1 f'(x) dx > \int_0^0 f'(x) dx$;
II is true since $\int_0^2 f'(x) dx > \int_0^1 f'(x) dx$ because $f(2)$ sweeps out more positive area than $f(1)$.
III is false since $f(3)$ adds just a small negative area to $f(2)$, giving a value that is still larger than $f(1)$.
7. D

Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t) dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about 3,000 barrels.



The Integral as an Accumulator Solutions

Free Response

1.

(a) $\int_0^6 R(t) dt = 31.815$ or 31.816 yd^3

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x)) dx$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or $-1.909 \text{ yd}^3/\text{hr}$

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.

The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

3 : $\begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.



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2.

(a) Method 1: $\int_0^3 \sqrt{t+1} dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0; \quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

(c) Method 1:

$$\begin{aligned} A(t) &= 30 + \int_0^t (8 - \sqrt{x+1}) dx \\ &= 30 + 8t - \int_0^t \sqrt{x+1} dx \end{aligned}$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for

$63 < t < 120$. Therefore there is a maximum

at $t = 63$.

Method 1:

$$3 \begin{cases} 2 : \text{definite integral} \\ 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

– or –

Method 2:

$$3 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$$

1 : answer

Method 1:

$$2 \begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1} dx \end{cases}$$

– or –

Method 2:

$$2 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$$

$$3 \begin{cases} 1 : \text{sets } A'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$$



The Integral as an Accumulator Solutions

3.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

2 : $\left\{ \begin{array}{l} 1 : \text{intervals} \\ 1 : \text{reason} \end{array} \right.$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

5 : $\left\{ \begin{array}{l} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{array} \right.$

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.