AP Calculus BC Saturday Study Session #2: Tabular Data Problems

Multiple Choice Solutions

1. D
2. A
3. B
4. A
5. E
6. D
7. E
8. C
9. B

Free Response Solutions

❶ 2012 #1 (BC) – Calc OK – Scoring Guidelines:

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$

$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately $1.017\ ^\circ\text{F}$ per minute at time $t = 12$ minutes.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\, dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by $16\ ^\circ\text{F}$ over the interval from $t = 0$ to $t = 20$ minutes.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

(c) $\dfrac{1}{20}\int_0^{20} W(t)\, dt \approx \dfrac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$

$= \dfrac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$

$= \dfrac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\, dt$

$= 71.0 + 2.043155 = 73.043$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

❷ 2011B #5 (BC) – No Calc – Scoring Guidelines:

(a) $a(5) \approx \frac{v(10) - v(0)}{10 - 0} = \frac{0.3}{10} = 0.03 \text{ meters/sec}^2$

1 : answer

(b) $\int_0^{60} |v(t)|\, dt$ is the total distance, in meters, Ben rides over the 60-second interval $t = 0$ to $t = 60$.

$\int_0^{60} |v(t)|\, dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139$ meters

$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{approximation} \end{cases}$

(c) Because $\frac{B(60) - B(40)}{60 - 40} = \frac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $2L(t)L'(t) = 2B(t)B'(t)$

$L'(40) = \frac{B(40)v(40)}{L(40)} = \frac{9 \cdot 2.5}{\sqrt{144 + 81}} = \frac{3}{2}$ meters/sec

$3 : \begin{cases} 1 : \text{derivatives} \\ 1 : \text{uses } B'(t) = v(t) \\ 1 : \text{answer} \end{cases}$

1 : units in (a) or (b)

❸ 2010B #3 (BC) – Calc OK – Scoring Guidelines:

(a) $\int_0^{12} P(t)\, dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t)\, dt = 225.594 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t)\, dt - \int_0^{12} R(t)\, dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

1 : answer

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\left.\frac{dh}{dt}\right|_{t=8} = \frac{1}{144\pi} \cdot \left.\frac{dV}{dt}\right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left.\frac{dh}{dt}\right|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$

❹ 2009 #5 (BC) – No Calc – Scoring Guidelines:

(a) $f'(4) \approx \dfrac{f(5)-f(3)}{5-3} = -3$	1 : answer
(b) $\int_2^{13} (3 - 5f'(x))\,dx = \int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$ $= 3(13-2) - 5(f(13) - f(2)) = 8$	2 : { 1 : uses Fundamental Theorem of Calculus; 1 : answer }
(c) $\int_2^{13} f(x)\,dx \approx f(2)(3-2) + f(3)(5-3)$ $+ f(5)(8-5) + f(8)(13-8) = 18$	2 : { 1 : left Riemann sum; 1 : answer }
(d) An equation for the tangent line is $y = -2 + 3(x-5)$. Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \le 8$. Therefore, $f(7) \le -2 + 3 \cdot 2 = 4$. An equation for the secant line is $y = -2 + \frac{5}{3}(x-5)$. Since $f''(x) < 0$ for all x in the interval $5 \le x \le 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$. Therefore, $f(7) \ge -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.	4 : { 1 : tangent line; 1 : shows $f(7) \le 4$; 1 : secant line; 1 : shows $f(7) \ge \frac{4}{3}$ }

❺ 2007 #5 (BC) – No Calc – Scoring Guidelines:

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.	$2: \begin{cases} 1: \text{estimate} \\ 1: \text{conclusion with reason} \end{cases}$
(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$ $\left.\frac{dV}{dt}\right\vert_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$	$3: \begin{cases} 2: \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$ $= 19.3$ ft $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.	$2: \begin{cases} 1: \text{approximation} \\ 1: \text{explanation} \end{cases}$
(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.	1 : conclusion with reason
Units of ft^3/min in part (b) and ft in part (c)	1 : units in (b) and (c)

❻ 2012 #4 (BC) a,b – No Calc – Scoring Guidelines:

(a) $f(1) = 15$, $f'(1) = 8$ An equation for the tangent line is $y = 15 + 8(x - 1)$. $f(1.4) \approx 15 + 8(1.4 - 1) = 18.2$	$2: \begin{cases} 1: \text{tangent line} \\ 1: \text{approximation} \end{cases}$
(b) $\int_1^{1.4} f'(x)\,dx \approx (0.2)(10) + (0.2)(13) = 4.6$ $f(1.4) = f(1) + \int_1^{1.4} f'(x)\,dx$ $f(1.4) \approx 15 + 4.6 = 19.6$	$3: \begin{cases} 1: \text{midpoint Riemann sum} \\ 1: \text{Fundamental Theorem of Calculus} \\ 1: \text{answer} \end{cases}$

❼ 2005 #3 (BC) – Calc OK – Scoring Guidelines:

(a) $\dfrac{T(8)-T(6)}{8-6} = \dfrac{55-62}{2} = -\dfrac{7}{2}\,^\circ\text{C/cm}$

1 : answer

(b) $\dfrac{1}{8}\int_0^8 T(x)\,dx$

Trapezoidal approximation for $\int_0^8 T(x)\,dx$:

$$A = \frac{100+93}{2}\cdot 1 + \frac{93+70}{2}\cdot 4 + \frac{70+62}{2}\cdot 1 + \frac{62+55}{2}\cdot 2$$

Average temperature $\approx \dfrac{1}{8}A = 75.6875^\circ\text{C}$

3 : { 1 : $\frac{1}{8}\int_0^8 T(x)\,dx$; 1 : trapezoidal sum; 1 : answer }

(c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45^\circ\text{C}$

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

2 : { 1 : value; 1 : meaning }

(d) Average rate of change of temperature on $[1, 5]$ is $\dfrac{70-93}{5-1} = -5.75$.

Average rate of change of temperature on $[5, 6]$ is $\dfrac{62-70}{6-5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in $[0, 8]$.

2 : { 1 : two slopes of secant lines; 1 : answer with explanation }

Units of $^\circ\text{C/cm}$ in (a), and $^\circ\text{C}$ in (b) and (c)

1 : units in (a), (b), and (c)