AP Calculus BC Saturday Study Session #2: Particle Motion

Multiple Choice Solutions

1. A
2. C
3. B
4. A
5. D
6. E
7. C
8. B
9. C
10. D
11. C
12. E
13. B
14. B
15. D
16. C
17. C
18. E

Free Response Solutions

❶ 2010B #4 (BC) – No Calc – Scoring Guidelines:

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

$2: \begin{cases} 1: t\text{-values} \\ 1: \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

$2: \begin{cases} 1: \text{identifies candidates} \\ 1: \text{answers} \end{cases}$

(c) The total distance traveled is $\int_0^{18} |v(t)|\, dt = 140 + 50 + 25 = 215.$

1 : answer

(d) For $7 < t < 10$, $a(t) = \frac{20 - (-10)}{7 - 10} = -10$

$v(t) = 20 - 10(t - 7) = -10t + 90$

$x(7) = \frac{7+5}{2} \cdot 20 = 120$

$$x(t) = x(7) + \int_7^t (-10u + 90)\, du$$

$$= 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t}$$

$$= -5t^2 + 90t - 265$$

$4: \begin{cases} 1: a(t) \\ 1: v(t) \\ 2: x(t) \end{cases}$

❷ 2009 #1 (BC) – Calc OK – Scoring Guidelines:

(a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{units} \end{cases}$
(b) $\int_0^{12} \lvert v(t) \rvert \, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$. $$\int_0^{12} \lvert v(t) \rvert \, dt = \int_0^2 v(t)\,dt - \int_2^4 v(t)\,dt + \int_4^{12} v(t)\,dt = 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$	$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{value of integral} \end{cases}$
(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$
(d) $\int_0^{12} w(t)\,dt = 1.6$; Larry lives 1.6 miles from school. $\int_0^{12} v(t)\,dt = 1.4$; Caren lives 1.4 miles from school. Therefore, Caren lives closer to school.	$3 : \begin{cases} 2 : \text{Larry's distance from school} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{value} \\ 1 : \text{Caren's distance from school} \\ \quad \text{and conclusion} \end{cases}$

❸ 2008 #4 (BC) – No Calc – Scoring Guidelines:

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$ $$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.	$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t)\,dt \\ 1 : \text{conclusion} \end{cases}$
(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.	$3 : \begin{cases} 1 : \text{positions at } t = 3,\ t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$
(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.	1 : answer with reason
(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

❹ 2012 #2 (BC) – Calc OK – Scoring Guidelines:

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Because $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\left.dy/dt\right|_{t=2}}{\left.dx/dt\right|_{t=2}} = 3.055 \text{ (or } 3.054)$

$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253 \text{ (or } 1.252)$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575 \text{ (or } 0.574)$

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

❺ 2010 #3 (BC) – Calc OK – Scoring Guidelines:

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second

1 : answer

(b) $x'(t) = 2t - 4$

Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\, dt = 11.587$ or 11.588 meters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \neq 0$

This occurs at $t = 2.20794$.

Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.

$3 : \begin{cases} 1 : \text{considers } \frac{dy}{dx} = 0 \\ 1 : t = 2.207 \text{ or } 2.208 \\ 1 : \text{direction of motion with reason} \end{cases}$

(d) $x(t) = 5$ at $t = 1$ and $t = 3$

At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = 0.432$.

At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3} = 1$.

$y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\, dt = 4$

$3 : \begin{cases} 1 : t = 1 \text{ and } t = 3 \\ 1 : \text{slopes} \\ 1 : y\text{-coordinate} \end{cases}$

❻ 2010B #2 (BC) – Calc OK – Scoring Guidelines:

(a) The tangent line is vertical when $x'(t) = 0$ and $y'(t) \neq 0$.

On $0 < t < 1.5$, this happens at $t = 1.253$ and $t = 1.144$ or 1.145.

$$2 : \begin{cases} 1 : \text{sets } \dfrac{dx}{dy} = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $\left.\dfrac{dy}{dx}\right|_{t=1} = \dfrac{y'(1)}{x'(1)} = 0.863447$

$x(1) = -2 + \int_0^1 x'(t)\,dt = 9.314695$

$y(1) = 3 + \int_0^1 y'(t)\,dt = 4.620537$

The line tangent to the path of the particle at $t = 1$ has equation $y = 4.621 + 0.863(x - 9.315)$.

$$4 : \begin{cases} 1 : \left.\dfrac{dy}{dx}\right|_{t=1} \\ 1 : x(1) \\ 1 : y(1) \\ 1 : \text{equation} \end{cases}$$

(c) Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$

1 : answer

(d) Acceleration vector: $\langle x''(1), y''(1) \rangle = \langle -28.425, 2.161 \rangle$

$$2 : \begin{cases} 1 : x''(1) \\ 1 : y''(1) \end{cases}$$