Euler's Method & Logistic Growth Functions Solutions

I. Euler's Method - Multiple Choice

1.

Original x	Original y	$\Delta x = dx$	$\Delta y \approx dy = (x+y)*dx$	New x = Original x+ dx	New y = Original y + dy
1	2	0.5	(1+2)*(0.5)=1.5	1.5	3.5
1.5	3.5	0.5	(1.5+3.5)*(0.5)=2.5	2.0	6.0

C

2.

Original x	Original y	$\Delta x = dx$	$\Delta y \approx dy = (2x+y)*dx$	New x = Original x+ dx	New y = Original y + dy
1	-3	0.5	(2*1+(-3))*(0.5)=-0.5	1.5	-3.5
1.5	-3.5	0.5	(2*1.5+(-3.5))*(0.5)=-0.25	2.0	-3.75

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II. Euler's Method – Free Response

1.

(b) $f(1.5) \approx f(1) + f'(1)(0.5)$

$= 4 - 3(1)(4)(0.5) = -2$

$f(2) \approx -2 + f'(1.5)(0.5)$

$\approx -2 - 3(1.5)(-2)(0.5) = 2.5$

2 : { 1 : Euler's method equations or equivalent table
1 : Euler approximation to $f(2)$ (not eligible without first point) }

2.

(c) $f\left(\frac{1}{2}\right) \approx f(0) + f'(0)\cdot\frac{1}{2} = -2 + (-3)\cdot\frac{1}{2} = -\frac{7}{2}$

$f'\left(\frac{1}{2}\right) \approx 3\left(\frac{1}{2}\right) + 2\left(-\frac{7}{2}\right) + 1 = -\frac{9}{2}$

$f(1) \approx f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right)\cdot\frac{1}{2} = -\frac{7}{2} + \left(-\frac{9}{2}\right)\cdot\frac{1}{2} = -\frac{23}{4}$

2 : { 1 : Euler's method with 2 steps
1 : Euler's approximation for $f(1)$ }

(d) $g'(0) = 3\cdot 0 + 2\cdot k + 1 = 2k + 1$

$g(1) \approx g(0) + g'(0)\cdot 1 = k + (2k+1) = 3k + 1 = 0$

$k = -\frac{1}{3}$

2 : { 1 : $g(0) + g'(0)\cdot 1$
1 : value of k }

Euler's Method & Logistic Growth Functions Solutions

III. Logistic Functions – Multiple Choice

1.

$$\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$$

$$or$$

$$\frac{dM}{dt} = \frac{0.6}{200}M(200 - M)$$

$$\Rightarrow M = 200$$

$$\Rightarrow B$$

2.

$$M = 200$$

$$\frac{dP}{dt} = kP(200 - P) = 200kP - kP^2$$

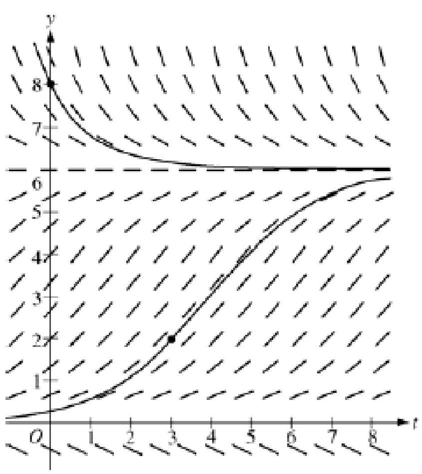
$$\Rightarrow k = 0.001$$

$$\Rightarrow A$$

IV. Logistic Functions – Free Response

1.

(a)



$2 : \begin{cases} 1: \text{solution curve through } (0,8) \\ 1: \text{solution curve through } (3,2) \end{cases}$

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2.

(a) For this logistic differential equation, the carrying capacity is 12. If $P(0) = 3$, $\lim_{t \to \infty} P(t) = 12$. If $P(0) = 20$, $\lim_{t \to \infty} P(t) = 12$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{answer} \end{cases}$
(b) The population is growing the fastest when P is half the carrying capacity. Therefore, P is growing the fastest when $P = 6$.	1 : answer