The Second Derivative

Since the derivative is itself a function, we can calculate its derivative. This is called the **second derivative** and, as we will see, it also gives us useful information about the original function.

First, let's look at some notation. Let y = f(x).

First derivative: Second derivative:

(see next slide)

The function given by $a(t)$ is the **second derivative** of $s(t)$ and is denoted by $s''(t)$.

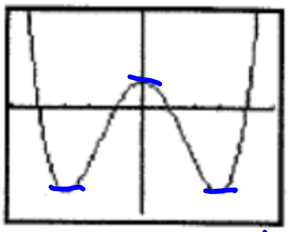
The second derivative is an example of a **higher-order derivative.** You can define derivatives of any positive integer order. For instance, the **third derivative** is the derivative of the second derivative. Higher-order derivatives are denoted as follows.

First derivative:	y',	$f'(x)$,	$\frac{dy}{dx}$,	$\frac{d}{dx}[f(x)]$,	$D_x[y]$
Second derivative:	y'',	$f''(x)$,	$\frac{d^2y}{dx^2}$,	$\frac{d^2}{dx^2}[f(x)]$,	$D_x^2[y]$
Third derivative:	y''',	$f'''(x)$,	$\frac{d^3y}{dx^3}$,	$\frac{d^3}{dx^3}[f(x)]$,	$D_x^3[y]$
Fourth derivative:	$y^{(4)}$,	$f^{(4)}(x)$,	$\frac{d^4y}{dx^4}$,	$\frac{d^4}{dx^4}[f(x)]$,	$D_x^4[y]$
$\vdots$					
nth derivative:	$y^{(n)}$,	$f^{(n)}(x)$,	$\frac{d^ny}{dx^n}$,	$\frac{d^n}{dx^n}[f(x)]$,	$D_x^n[y]$

Question: What information does the second derivative tell us?

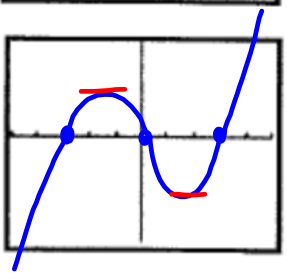
Let's look at this graphically with an example.

This is the graph of a function f.



Sketch the graph of the derivative of f, f′.

f′



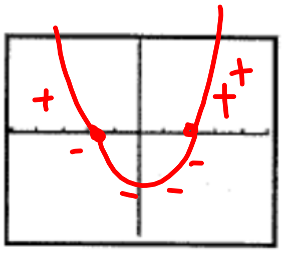
Recall the information that the derivative tells us about the derivative of a function.

On an interval,

- if f′ > 0, then f is increasing
- if f′ < 0, then f is decreasing

Sketch the graph of the derivative of f′, f″.

f″



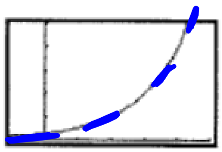
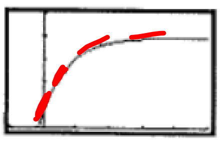
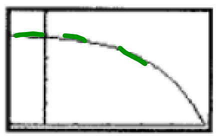
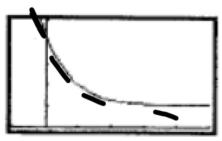
Since f″ is the derivative of f′, we have:
On an interval,

- if f″ > 0, then f′ is increasing
- if f′ < 0, then f′ is decreasing

So, the question now becomes: **What does it mean for f′ to be increasing or decreasing?**

Look at the graphs of four functions (f) below. For each graph, determine f′ is **increasing** or **decreasing**?
Remember, f′ represents the <u>slope of the curve</u> (or the <u>slope of the tangent line</u>).

f

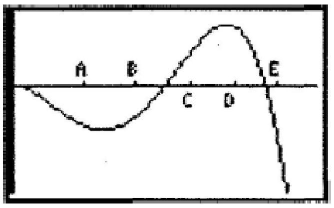
f′ is increasing	f′ is decreasing	f′ is decreasing	f′ is increasing
f is concave up	f is concave down	f is concave down	f is concave up
½ 1 5 10	10 5 1 ½	-½ -1 -5 -10	-10 -5 -1 -½

Conclusions: On an interval,

f'' > 0 means f' is increasing	, so f is concave up
f'' < 0 means f' is decreasing	, so f is concave down

Examples:

1. Given the graph of a function f below. At which marked x-value(s) are the following statements true?



a. f(x) < 0 ________ d. f(x) is inc ________

b. f'(x) < 0 ________ e. f'(x) is inc ________

c. f''(x) < 0 ________

$$f(x)=\frac{x^2+2x-1}{x}, \qquad \textit{find} \quad f''(x)$$

$$f(x)=\frac{x^2}{x}+\frac{2x}{x}-\frac{1}{x}$$

$$f(x)=x+2-x^{-1}$$

$$f'(x)=1+x^{-2}$$

$$f''(x)=-2x^{-3}$$

If the nth derivative of y is denoted as y^n and y = -sin x, then y^7 is the same as:

(A) y (B) $\frac{dy}{dx}$ (C) $\frac{d^2y}{dx^2}$ (D) $\frac{d^3y}{dx^3}$ (E) none of the above

$y = -\sin x$ $\quad y^4 = -\sin x$

$y' = -\cos x$ $\quad y^5$

$y'' = \sin x$ $\quad y^6$

$y''' = \cos x$ $\quad y^7$

$y^4 = -\sin x$

Stand and Deliver

2.3

Higher Order Derivatives

s(t) = position

v(t) = s'(t) = velocity

speed = $|v(t)|$

a(t) = v'(t) = s''(t) = acceleration

look at additional notation on p. 125

Position Function

$$s(t) = \frac{1}{2}gt^2 + v_o t + s_o$$

v_o = initial velocity

s_o = initial height

t = time

g = gravity

- 32 ft/sec^2

or

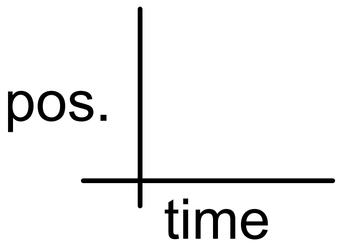
- 9.8 m/sec^2

Average Velocity vs. Instantaneous Velocity

Average Velocity

Algebra slope
(no calculus needed)

$$\frac{\Delta s}{\Delta t} = \frac{\text{position}}{\text{time}}$$



Velocity

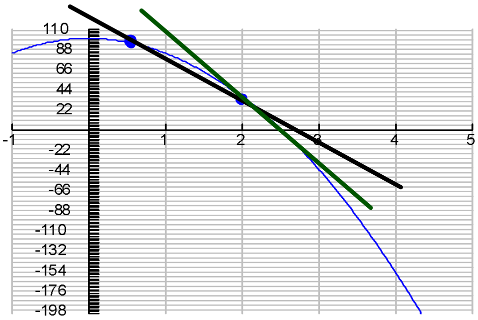
Instantaneous Velocity

velocity at a point

$s'(t) = v(t)$

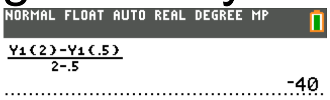
take the derivative

$s(t) = -16t^2 + 100$



a) Find the average velocity from [0.5, 2]

$(.5, s(.5))\ (2, s(2))$



b) Find the instantaneous velocity at t = 2

$s'(t) = -32t$

$s'(2) = -32(2) = -64$

no calculator

A penny falling is given by the function

$s(t) = -16t^2 + 32t + 48$

a) When does it hit the ground?

$\frac{0}{-16} = \frac{-16t^2 + 32t + 48}{-16}$

$0 = t^2 - 2t - 3$

$0 = (t-3)(t+1)$

$t = 3, \cancel{-1}$

b) What is the velocity when it hits the ground?

$s'(3) = ?$

$s'(t) = -32t + 32$

$s'(3) = -32(3) + 32$

$s'(3) = -64$