

Graphs of  $f$  and  $f'$

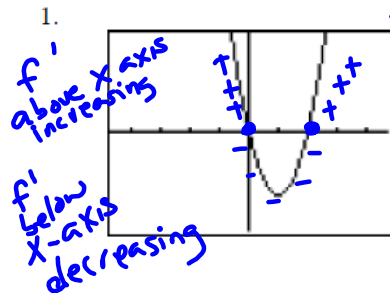
1. In the left column below are graphs of several functions. In the right-hand column - in a different order - are graphs of the associated derivative functions. Match each function with its derivative. (Note: The scales on the graphs are not all the same.)

	a. <sup>iii</sup> 	i. 	
	b. <sup>i</sup> 	ii. 	
	c. <sup>ii</sup> 	iii. 	
	d. <sup>i</sup> 	iv. 	
	e. <sup>iv</sup> 	v. 	

- 2.(a) Sketch a graph of the derivative of each function labeled (i) - (v) in the right column of the preceding problem.
- (b) (Optional!) For each function labeled (a) - (e) in the left column of the preceding problem, sketch a graph of a function whose derivative is the function shown.

The second question we wanted to answer was: **What does the derivative  $f'$  tell us about  $f$ ?**

Examples: Below is the graph of  $f'$ , the derivative of a function  $f$ .



On what interval(s) is the function  $f$

a. increasing?  $(-\infty, 0) \cup (2, \infty)$  *Where  $f'$  is positive*

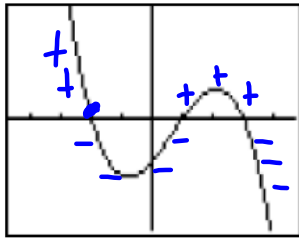
b. decreasing?  $(0, 2)$  *Where  $f'$  is negative*

At what x-value does  $f$  have a

c. maximum? 0      d. minimum? 2

*Where  $f$  changes from increasing to decreasing*  
 or *Where  $f'$  changes from positive to negative*

2.



On what interval(s) is the function f

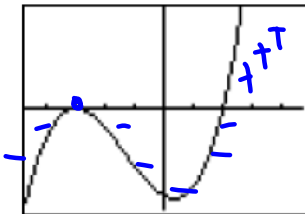
a. increasing?  $(-\infty, -2)$   $(2, 3)$

b. decreasing?  $(-2, 1)$   $(3, \infty)$

At what x-value does f have a

c. maximum?  $-2, 3$  d. minimum?  $1$

3.



On what interval(s) is the function f

a. increasing?  $(2, \infty)$

b. decreasing?  $(-\infty, -3)$   $(-3, 2)$

At what x-value does f have a

c. maximum?  $-$  d. minimum?  $2$

function

Using table values and applying derivative rules pg. 2

Two functions,  $f(x)$  and  $g(x)$ , are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are given in the following table.

$x$	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Based on the table, calculate the following:

(a)  $\frac{d}{dx}(f(x) + g(x))$ , evaluated at  $x = 4$

$f'(4) + g'(4) = -\frac{4}{5} + -\frac{1}{3} = -\frac{12}{15} - \frac{5}{15} = -\frac{17}{15}$

$3 + -\frac{1}{3} = 2\frac{2}{3}$

(b)  $\frac{d}{dx}(f(x)g(x))$ , evaluated at  $x = 1$

Product Rule

$f(1)g'(1) + g(1)f'(1) = (\frac{1}{3})(\frac{2}{3}) + (1)(\frac{5}{3}) = \frac{2}{9} + \frac{5}{3} = \frac{2}{9} + \frac{15}{9} = \frac{17}{9}$

(c)  $\frac{d}{dx}(\frac{f(x)}{g(x)})$ , evaluated at  $x = 0$

Quotient Rule

$\frac{g(0)f'(0) - f(0)g'(0)}{(g(x))^2} = \frac{-2(\frac{3}{2}) - (\frac{1}{2})(-1)}{4} = \frac{-3 + \frac{1}{2}}{4} = \frac{-\frac{5}{2}}{4} = -\frac{5}{8}$

(d)  $\frac{d}{dx}(f(g(x)))$ , evaluated at  $x = 3$

Chain Rule

$f'(g(3)) \cdot g'(3)$

$f'(2) \cdot -3$

$\frac{1}{4} \cdot -3 = -\frac{3}{4}$

~~$(0) \cdot (-3) = 0$~~

$x$	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$