The Derivative Function Graphically

Yesterday, we looked at the derivative of a function at a point. We also saw that the derivative takes on different values at different points and is itself a function.

Remember, **the derivative f'(a) is the slope of the tangent line to the graph of f at x = a.**

One question we want to answer now is:

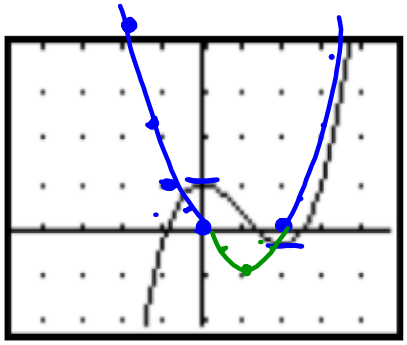
Given the graph of a function f, can we sketch a graph of the derivative function f'?

And another question is:

What does the derivative f' tell us about f?

We are going to try to answer these questions, because, if we can, we should have a good understanding of the derivative function!

Example: Below is the graph of a function f.



a. At what x-value(s) does it appear that the derivative of f is zero? (slope)

x = 0, 2

b. At what x-values is f increasing,? (-∞,0) (2,∞)

What does this tell us about f'? positive

*if function is increasing, derivative is positive

c. At what x-values is f decreasing,? (0,2)

What does this tell us about f'? negative

d. On the graph of, sketch a graph of f'

Slope

big positive

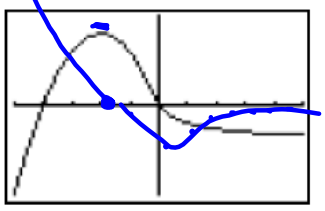
small positive

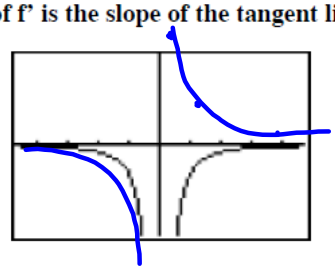
big negative

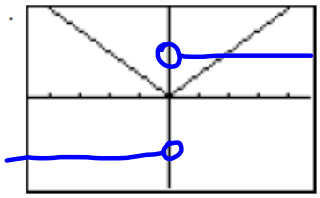
small negative

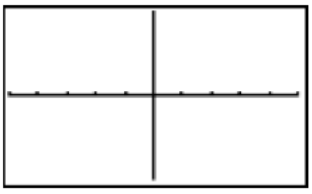
More Examples: Given the graph of the function shown, sketch the graph of the derivative function f' directly below it.

Remember, **"the y value on the graph of f' is the slope of the tangent line to the graph of f."**

1. 

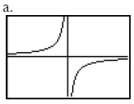
2 

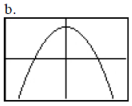
3. . 

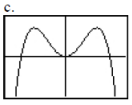


Graphs of f and f'

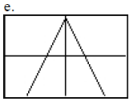
1. In the left column below are graphs of several functions. In the right-hand column - in a different order - are graphs of the associated derivative functions. Match each function with its derivative. (Note: The scales on the graphs are not all the same.)

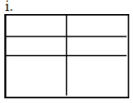
a. 

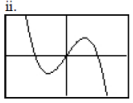
b. 

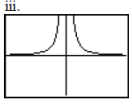
c. 

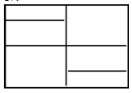
d. 

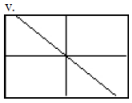
e. 

i. 

ii. 

iii. 

iv. 

v. 

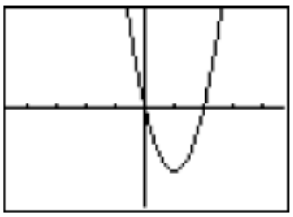
2.(a) Sketch a graph of the derivative of each function labeled (i) - (v) in the right column of the preceding problem.

(b) (Optional!) For each function labeled (a) - (e) in the left column of the preceding problem, sketch a graph of a function whose derivative is the function shown.

The second question we wanted to answer was: **What does the derivative f' tell us about f?**

Examples: Below is the graph of f', the derivative of a function f.

1.



On what interval(s) is the function f

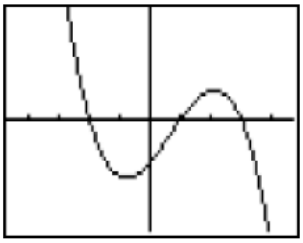
a. increasing? _____

b. decreasing? _____

At what x-value does f have a

c. maximum? _____ d. minimum? _____

2.



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At what x-value does f have a

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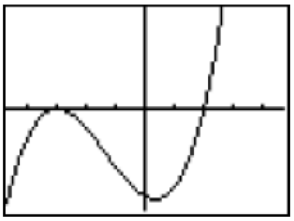
3.



On what interval(s) is the function f

a. increasing? _____

b. decreasing? _____

At what x-value does f have a

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Using table values and applying derivative rules pg. 2

Two functions, $f(x)$ and $g(x)$, are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are given in the following table.

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Based on the table, calculate the following:

(a) $\frac{d}{dx}(f(x)+g(x))$, evaluated at $x = 4$

$f'(4)+g'(4)$

(b) $\frac{d}{dx}(f(x)g(x))$, evaluated at $x = 1$ product Rule

(c) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$, evaluated at $x = 0$ Quotient

(d) $\frac{d}{dx}(f(g(x)))$, evaluated at $x = 3$ Chain

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$