Objective: You will be able to:

- find the derivative using the Quotient Rule
- find the derivative of composite functions

Stand and Deliver

Calculus the Musical: Quotient Rule

Quotient Rule 2.3

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

"lo D hi, minus hi D lo, over lo lo"

Examples

1. $g(x) = \dfrac{3x - 1}{x^2 + 3}$

$$g'(x) = \frac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)^2}$$

$$g'(x) = \frac{3x^2+9-6x^2+2x}{(x^2+3)^2}$$

$$g'(x) = \frac{-3x^2+2x+9}{(x^2+3)^2}$$

trick question
don't need quotient rule

2. $y = \dfrac{x^2 + 3x}{6}$

$$y = \frac{1}{6}(x^2+3x) \longrightarrow \frac{1}{6}x^2 + \frac{x}{2}$$

$$y' = \frac{1}{6}(2x+3) = \frac{2x+3}{6}$$

y

Stand and Deliver

Calculus the Musical: Chain Rule

2.4

Chain Rule

$$\frac{d}{dx}[f(g(x)] = f'(g(x))\ g'(x)$$

"derivative of the outer (skin) with the inner (guts) in tact, times the derivative of the inner (guts)"

Ex. 1

$$y = (x^2 + 1)^3$$

$$\frac{dy}{dx} = 3(x^2+1)^2 \cdot (2x)$$

$$\frac{dy}{dx} = 6x(x^2+1)^2$$

maybe skip

Ex. 2

$$f(x) = (2x - 4x^2)^4$$

$$f'(x) = 4(2x-4x^2)^3 \cdot (2-8x)$$

$$f'(x) = 4(2-8x)(2x-4x^2)^3$$

Ex. 3

$$y = \sqrt{x^2 - 3x} = (x^2-3x)^{1/2}$$

$$y' = \frac{1}{2}(x^2-3x)^{-1/2} \cdot (2x-3)$$

$$y' = \frac{2x-3}{2(x^2-3x)^{1/2}} = \frac{2x-3}{2\sqrt{x^2-3x}}$$

Ex. 4

$$y = \frac{3}{(x-4x^3)^2} = 3(x-4x^3)^{-2}$$

$$y' = -6(x-4x^3)^{-3} \cdot (1-12x^2)$$

$$= \frac{-6(1-12x^2)}{(x-4x^3)^3}$$

Ex. 5

$$f(x) = \sin(3x) \qquad \neq \cancel{3\sin x}$$

$$f'(x) = \cos(3x)\cdot(3)$$

$$= 3\cos(3x)$$

Ex. 6

$$y = \tan(2x^2 - 1)$$

$$\frac{dy}{dx} = \sec^2(2x^2-1) \cdot (4x)$$

$$\frac{dy}{dx} = 4x \sec^2(2x^2-1)$$

Ex. 7

$$y = \cos^4 x = ((\cos x))^4$$

$$y' = 4(\cos x)^3 \cdot (-\sin x)$$

$$y' = -4 \sin x \cos^3 x$$

Ex. 8

$f(g(h(x)))$

$$f(x) = \sin^5(2t)$$

$$f(x) = (\sin(2t))^5$$

$$f'(x) = 5(\sin(2t))^4 \cdot \cos(2t) \cdot 2$$

$$= 10\cos(2t)\sin^4(2t)$$

Ex. 9

maybe skip

$$f(x) = \cot^3(x^2 - 5x)$$

$$f(x) = (\cot(x^2-5x))^3$$

$$f'(x) = 3(\cot(x^2-5x))^2 \cdot (-\csc^2(x^2-5x)) \cdot (2x-5)$$

$$= -3(2x-5)\cot^2(x^2-5x)\csc^2(x^2-5x)$$

Ex. 10 ★★★

see if you can match

$y' = x(x-3)^2(5x-6)$

$$y = \underbrace{x^2}_{1st}\underbrace{(x-3)^3}_{2nd}$$

Product Rule
chain Rule

$y' = x^2 \cdot 3(x-3)^2 \cdot (1) + (x-3)^3 \cdot 2x$

$y' = 3x^2(x-3)^2 + 2x(x-3)^3$

$x(x-3)^2[3x + 2(x-3)]$

$x(x-3)^2[3x+2x-6]$

$x(x-3)^2[5x-6]$