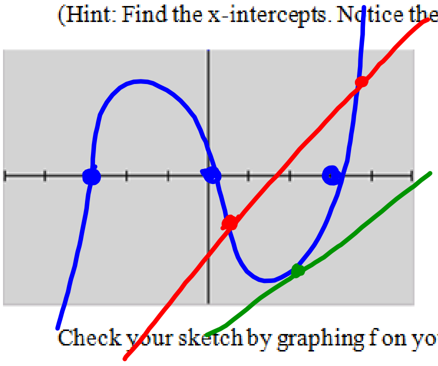
Average Rate of Change, Instantaneous Rate of Change, and the Definition of the Derivative

Example: Given $f(x) = x^3 - 9x$. (2 stand and deliver cards)

1. Can you sketch the graph of f without your calculator? (Sure you can!)

(Hint: Find the x-intercepts. Notice there is no y scale, just estimate)



$f(x) = x(x^2 - 9)$

$0 = x(x+3)(x-3)$

$x = 0, -3, 3$

Check your sketch by graphing f on your calculator.

a. Find **the average rate of change** of f from x = 1 to x = 4.

$f(1) = -8$

$f(4) = 4^3 - 36 = 64 - 36 = 28$

$\frac{f(4)-f(1)}{4-1} = \frac{28+8}{4-1} = \frac{36}{3}$

$= 12$

b. Write **the equation of the secant line** that passes through the function at x = 1 and x = 4. When finished, graph the secant line on your calculator.

$(1,-8)$ $m = 12$

$y+8 = 12(x-1)$

$y + 8 = 12x - 12 \rightarrow y = 12x - 20$

c. Estimate the **instantaneous rate of change** of f at x = 2.

$(2,-10)$ guess at the slope

d. Write the **equation of the tangent line** to f at x = 2. When finished, graph the tangent line on your calculator.

$m = 3$

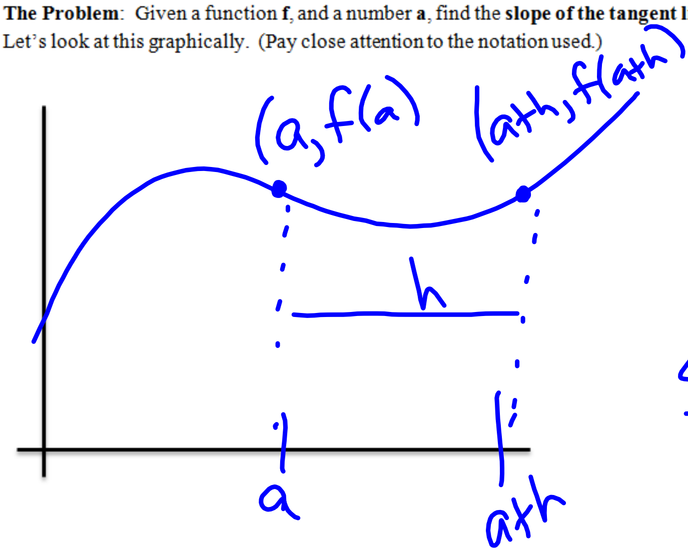
$y + 10 = 3(x-2)$

$y + 10 = 3x - 6$

$y = 3x - 16$

2. **The Tangent Line Problem (in general) and the Definition of the Derivative**

The Problem: Given a function **f**, and a number **a**, find the **slope of the tangent line** to f at **x** = **a**.
Let's look at this graphically. (Pay close attention to the notation used.)



as $h \to 0$
Slope of secant line
becomes
slope of
tangent line

Stand and Deliver

Calculus the Musical: Differientabul

2.1

Limit Definition to Find a Derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Example: If $f(x) = x^2 + 4x$, find the slope of the tangent line to f at a $a = 1$; i.e., find $f'(1)$.

$f'(1) =$

4. **Practice:** If $f(x) = x^2 + 4x$, use the definition of the derivative to find $f'(x)$ for $x = -5, -4, -3, -2, -1,$ *and* 0.

assign a number to a group

$f'(\_\_\_) =$

Place the values in the chart below.

x	-5	-4	-3	-2	-1	0	1
f(x)							6

Is there a pattern to these derivatives? In other words, is there a function rule that we can assign to $f'(x)$?

$f'(x) =$____________________________

So, the derivative of a function is also a function!

5. **Note**: Built into your calculator is a feature that will approximate derivatives of functions at a specific number.

To evaluate $f'(1)$ for $f(x)=x^2+4x$ in calculator…

6. **Definition of the Derivative Function** $f'(x)$.

Since, from #4 above, the derivative of a function appears to be a function itself, we can use the definition of the derivative at a number x = a, $f'(a)$, to obtain the definition of a derivative function $f'(x)$. This results in:

$f'(x)=$

7. Examples:

a. Use the definition of the derivative function $f'(x)$ to show that the derivative of $f(x)=x^2+4x$ is $f'(x)=2x+4$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

b. Use the definition of the derivative function $f'(x)$ to show that the derivative of $f(x) = -3x^2 - 5$

Stand and Deliver

2.1

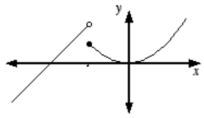
3 Scenarios when function is not differentiable

1. not continuous

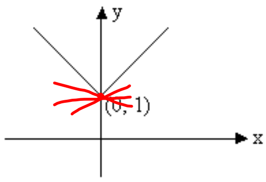
2. corner or cusp

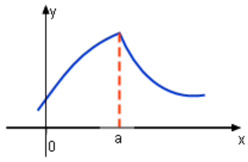
3. vertical tangent line

1. not continuous



2. corner or cusp





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3. vertical tangent line