Stand and Deliver

There are many important formulas and concepts you must MEMORIZE to get through Calculus.
So when you see one of these slides...
- a. Grab a notecard
- b. Copy the slide
- c. MEMORIZE!

When we play Stand and Deliver as a class...
it's just like it sounds...
when it's your turn, you must STAND
and DELIVER the formula!!

Stand and Deliver

PC

Pythagorean Identities

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 + \cot^2 x = \csc^2 x$

Stand and Deliver

PC

Double Angle

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$\underline{\cos^2 x - \sin^2 x}$

$1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x$

Stand and Deliver

PC

Power Reducing

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Stand and Deliver

PC

Unit Circle

	sin θ	cos θ	tan θ
0			
π/6			
π/4			
π/3			
π/2			

Zeno's paradoxes

Greek philosopher Zeno of Elea (ca. 490–430 BC)

Some mathematicians and historians, such as Carl Boyer, hold that Zeno's paradoxes are simply mathematical problems, for which modern calculus provides a mathematical solution.

https://www.youtube.com/watch?v=EfqVnj-sgcc

Greatest integer

$[\![1.46]\!] = 1$

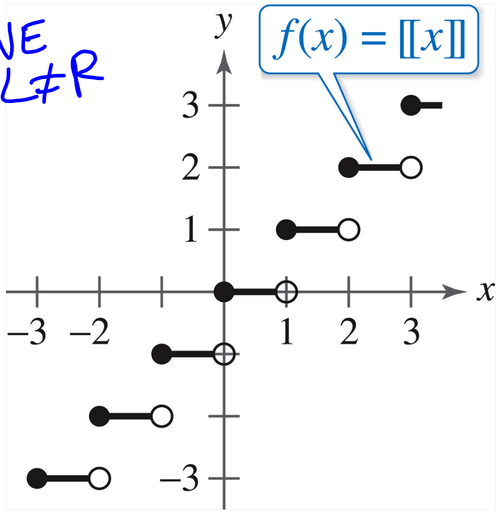
$[\![1.87]\!] = 1$

$[\![2.3]\!] = 2$

$$\lim_{x \to 2} [\![x]\!]$$

DNE

$L \neq R$

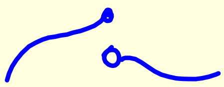


Stand and Deliver

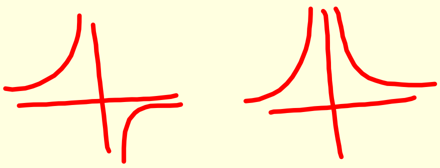
3 Ways Limits DNE

1.2

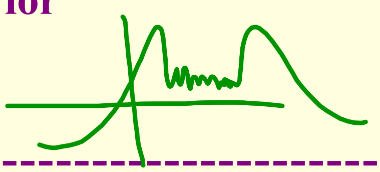
1. lim L $\neq$ lim R



2. unbound behavior



3. oscillating behavior

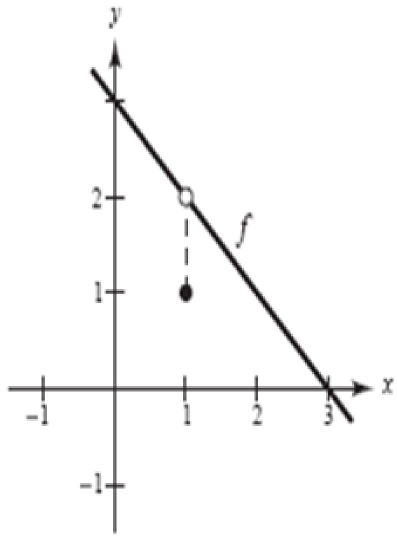


3. Use the graph to find $\lim_{x\to1} f(x)$ if $f(x) = \begin{cases} 3 - x, & x \neq 1 \\ 1, & x = 1 \end{cases}$.

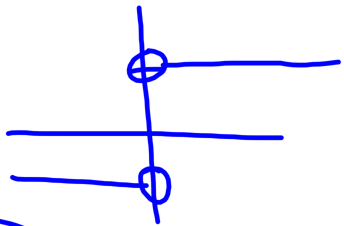
(a) 2 (b) 1

(c) $\frac{3}{2}$ (d) The limit does not exist.

(e) None of these



$$\lim_{x\to0}\frac{|x|}{x}$$



x	$y = \frac{\lvert x\rvert}{x}$
2	$\frac{\lvert 2\rvert}{2} = \frac{2}{2} = 1$
1	$\frac{\lvert 1\rvert}{1} = \frac{1}{1} = 1$
0	$\frac{\lvert 0\rvert}{0}$ = DNE
-1	$\frac{\lvert -1\rvert}{-1} = \frac{1}{-1} = -1$
-2	$\frac{\lvert -2\rvert}{-2} = \frac{2}{-2} = -1$

What is

(A) 0 (B) nonexistent (C) 1 (D) -1 (E) none of these

Stand and Deliver

Strategies for Finding Limits Algebraically 1.3

1. Try direct substitution
2. If direct substitution fails, try simplifying or factoring.
3. If direct substition fails, and there is a $\sqrt{\ }$ (multiply by the conjugate)

Limit of a Rational Function

1) $$\lim_{x \to 1} \frac{1-\sqrt{x}}{x-1}$$

2) $$\lim_{x \to 0} \frac{x^2-x}{x^4+x^3}$$

3) $$\lim_{x \to -4} \frac{x^3+2x^2-8x}{x^2+4x} =$$

4) $$\lim_{x \to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x} =$$

Limit of a Rational Function

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4) $\lim_{x \to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x} =$

1) $\lim_{x \to 1} \frac{1-\sqrt{x}}{x-1} = \frac{0}{0}$

$\lim_{x \to 1} \frac{(1-\sqrt{x})}{(x-1)} \cdot \frac{(1+\sqrt{x})}{(1+\sqrt{x})}$

$\lim_{x \to 1} \frac{1-x}{(x-1)(1+\sqrt{x})}$

$\lim_{x \to 1} \frac{-1}{1+\sqrt{x}}$

$\frac{-1}{1+\sqrt{1}} = \boxed{-\frac{1}{2}}$

2. $\lim_{x \to 0} \frac{x^2 - x}{x^4 + x^3}$ $\quad \frac{0}{0}$

$\lim_{x \to 0} \frac{x(x-1)}{x^3(x+1)}$

$\lim_{x \to 0} \frac{x-1}{x^2(x+1)}$

$\frac{-1}{0}$ DNE
unbound

3. $\lim_{x \to -4} \frac{x^3 + 2x^2 - 8x}{x^2 + 4x}$ $\quad \frac{0}{0}$

$\lim_{x \to -4} \frac{\cancel{x}(x^2 + 2x - 8)}{\cancel{x}(x+4)}$

$\lim_{x \to -4} \frac{\cancel{(x+4)}(x-2)}{\cancel{(x+4)}}$

$\lim_{x \to -4} x - 2$

-6

4. $$\lim_{x\to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x} \qquad \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\frac{3-(3+x)}{3(3+x)}}{x}$$

$$\lim_{x\to 0} \frac{\frac{-x}{3(3+x)}}{x}$$

$$\lim_{x\to 0} \frac{-x}{3(3+x)} \cdot \frac{1}{x}$$

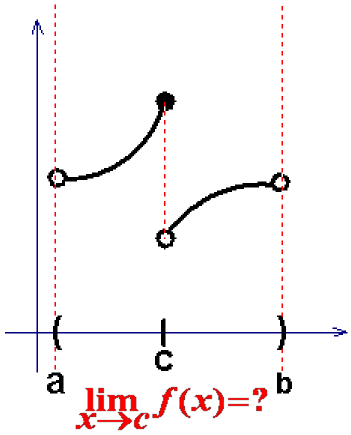
$$\lim_{x\to 0} -\frac{1}{3(3+x)}$$

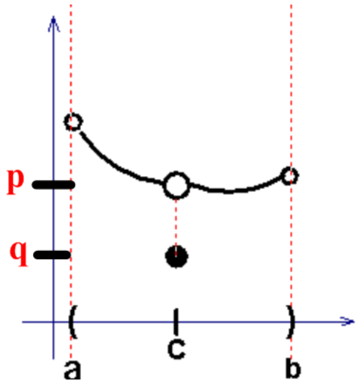
$$-\frac{1}{9}$$

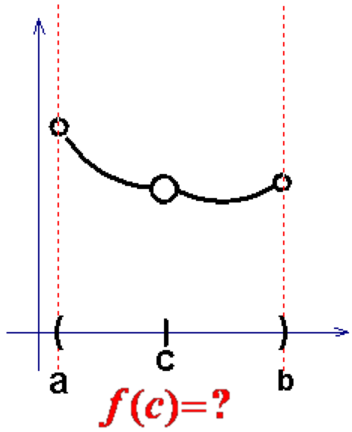
Stand and Deliver

A function is continuous at c if... 1.4

1. **f (c) is defined (the point exists)**
2. **$\lim_{x\to c} f(x)$ exists (the limit exists)**
3. **$f(c) = \lim_{x\to c} f(x)$ (the point = the limit)**







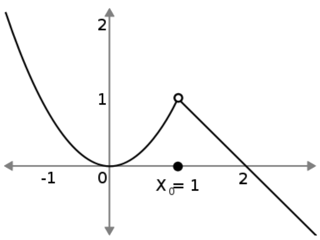
Classifying Discontinuities

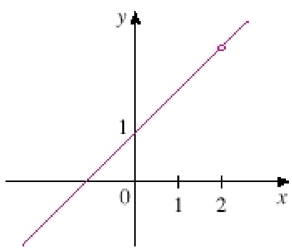
Removable or point
(Holes)
2 sided limit exists

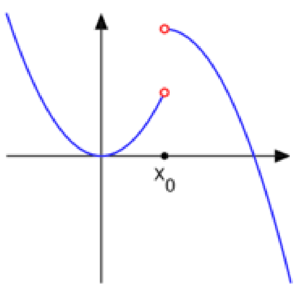
Essential or Non-removable

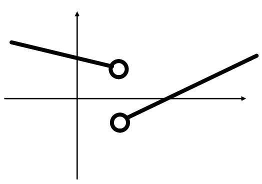
Jump
1 sided limits exist

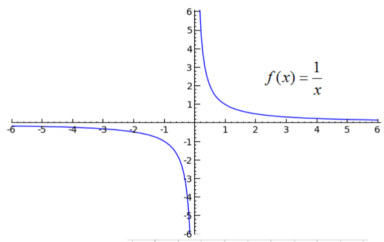
Infinite
(vertical asymptotes)
at least one of the
1 sided limits don't exist

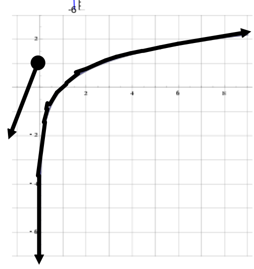






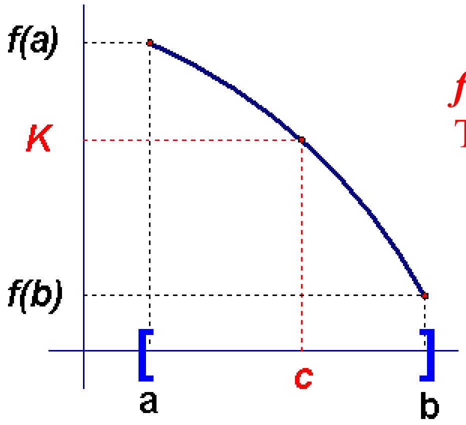






The Intermediate Value Theorem (page 77)

If f is continuous on the closed interval [a, b]
and k is any number between f(a) and f(b), then
there is at least one number c in [a, b]
such that f(c) = k.



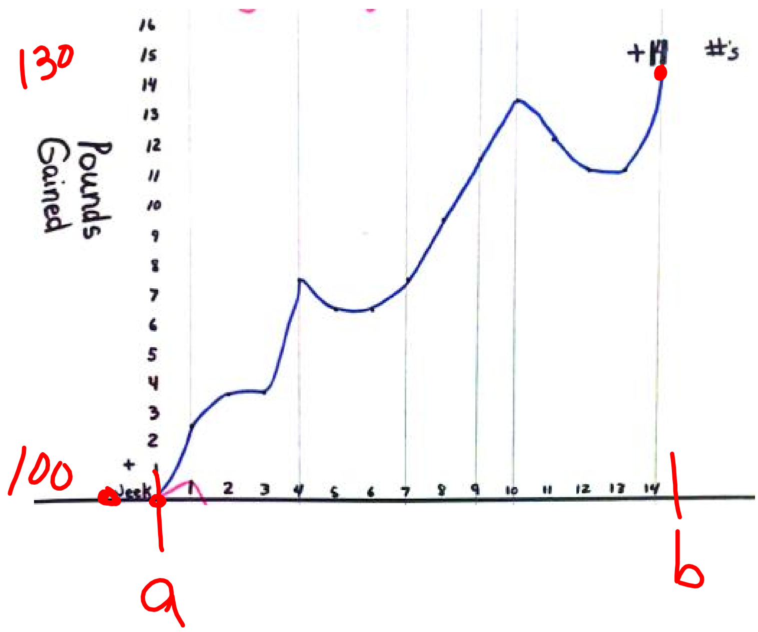
f is continuous on [a, b].
There exists a ***c*** such that f(c) = k.

http://www.calculus-help.com/tutorials

intermediate value theorem

(good)

need to use firefox

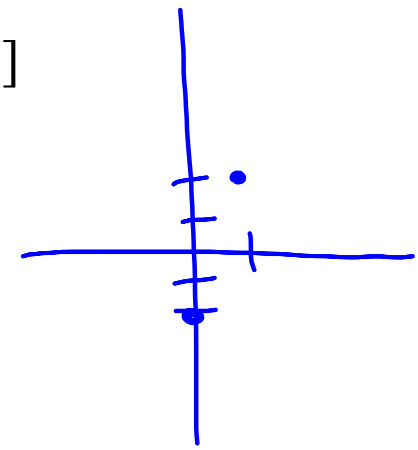


Explain why the function has a zero in the given interval:

$f(x) = x^3 + 3x - 2 \quad [0,1]$

$f(0) = -2$

$f(1) = 1^3 + 3(1) - 2$
$1 + 3 - 2$
$= 2$



After explaining to a student through various lessons and examples that:

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

I tried to check if she really understood that, so I gave her a different example. This was the result:

$$\lim_{x \to 5} \frac{1}{x-5} = 5$$

AP Limits Worksheet

Attachments

what is a limit.htm

Section_A__What_Is_Calculus_Anyway_.asf