Introduction to Limits

Both of the major calculus concepts - the derivative and the integral - rely on an understanding of the concept of a limit. So let's develop the concept of a limit. It's important that we understand this concept!

An important question: Given a function **f,** and a number **a**. As x gets closer and closer to **a**, but x does not equal **a,** does **f(x)** get closer and closer to some number **L**?

If it does, then we say "the limit of f(x), as x approaches a, is equal to L", and we write $\lim_{x \to a} f(x) = L$

Examples

1. Let $f(x) = 2x + 1$. As x gets closer and closer to some number, say 3, does f(x) get closer and closer to some value L? If it does, then we write $\lim_{x \to 3}(2x+1) = L$.

Let's see. Evaluate: $f(2.9) = 6.8$ $f(2.99) = 6.98$ $f(2.999) = 6.998$

$f(3.1) = 7.2$ $f(3.01) = 7.02$ $f(3.001) = 7.002$

So, the $\lim_{x \to 3}(2x+1) = 7$

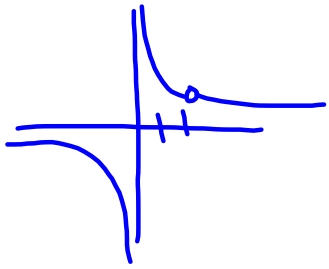
How else could we have evaluated this limit?

$2(3)+1=7$

2. Find $\lim_{x \to 2} \frac{x-2}{x^2-2x}$. Can we find the limit by substituting 2 for x? _____ Explain!

$\frac{2-2}{4-4} = \frac{0}{0}$ indeterminant form

Let's look at this problem graphically. Graph the function $f(x) = \frac{x-2}{x^2-2x}$. $= \frac{x-2}{x(x-2)} = \frac{1}{x}$



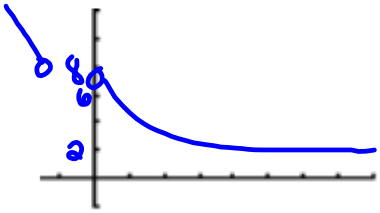
Evaluate the function close to x = 2 to determine the limit. Conclusion: $\lim_{x \to 2} \frac{x-2}{x^2-2x} = \frac{1}{2}$

How could we have evaluated this limit analytically?

$\lim_{x \to 2} \frac{x-2}{x^2-2x} =$ Factor & simplify

3. Given $f(x)=(1+x)^{2/x}$

a. Sketch a graph of the function for $x > -1$. Show all asymptotes with dotted lines and other undefined values with an "open circle".



b. Estimate the $\lim_{x\to 0} f(x)$ by evaluating f for values close to 0. Approximate the limit to 4 decimal places.

$\lim_{x\to 0}(1+x)^{2/x} \approx$ 7.3891

c. Do you know the **exact value** of $\lim_{x\to 0}(1+x)^{2/x}$? _____ Next semester you will

4. Analytically, find the following limit.

$$\lim_{x\to -3}\frac{x^2-9}{2x^2+5x-3}$$

$$\lim_{x\to -3}\frac{(x+3)(x-3)}{(2x-1)(x+3)}$$

$$\lim_{x\to -3}\frac{x-3}{2x-1}$$

$$\frac{-6}{-6-1}$$

$$\frac{6}{7}$$