Example 4

A particle moves in the xy-plane for t > 0 so that $\frac{dx}{dt}$

Given: $x(t) = t^2$ $x'(t) = 2t = \frac{dx}{dt}$

$y(t) = t^4 + 3t^2$ $y'(t) = 4t^3 + 6t$

Find: $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} =$

$\frac{d^2y}{dx^2}$

$$\frac{y'(t)}{x'(t)} = \frac{4t^3+6t}{2t} = \frac{\cancel{2t}(2t^2+3)}{\cancel{2t}} = 2t^2+3$$

$$\frac{dy}{dx} = 2t^2+3$$

$$\frac{d^2y}{dx^2} = 4t \cdot \frac{dt}{dx}$$

$$= \frac{4t}{\frac{dx}{dt}} = \frac{4t}{2t} = 2$$

# 3. Integrals and applications

- (total) distance traveled
- arc length

} same formula

- particle position

Example 1

A particle moves in the xy-plane so that its velocity for t > 0 is given by the parametric equations:

$$x'(t) = e^{2t}$$

$$y'(t) = \sqrt{3t+1}$$

Write the expression for the distance traveled by the particle on the time interval [1, 5]

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$\int_1^5 \sqrt{(e^{2t})^2 + (\sqrt{3t+1})^2}\,dt$$

NORMAL FLOAT AUTO REAL RADIAN MP

$\int_1^5 (\sqrt{Y_1^2+Y_2^2})dX$

11009.72322

Example 2

Find the length of the arc formed by the parametric equations on the interval $-2 \le t \le 1$

$x(t) = 2t+3$ $\quad x'(t) = 2$

$y(t) = \sqrt{t-2}$ $\quad y'(t) = \frac{1}{2}(t-2)^{-1/2}(1)$

$= (t-2)^{1/2}$ $\quad =$

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$\int_{-2}^1 \sqrt{(2)^2 + \left(\frac{1}{2}(t-2)^{-1/2}\right)^2}\,dt$$

$$\int_{-2}^1 \sqrt{4 + \frac{1}{4(t-2)}}\,dt$$

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$\int_{-2}^1 \left(\sqrt{4+\frac{1}{4(X-2)}}\right)dX$

5.912609471

$\approx 5.913$