Parametric Functions and Vector Functions

Despicable Me

Parametric:

1. Precalculus recap
2. Derivatives and applications
3. Integrals and applications

1. Precalculus recap

Eliminate the parameter

(write in rectangular or cartesian form)

2 types

- algebraic based
- trig based

Type 1

solve for t in one equation and sub into other...can you recognize the curve

$$x(t) = 2t - 5$$

$$y(t) = \frac{t^2}{3} + 1$$

$x = 2t - 5$

$\frac{x+5}{2} = \frac{\cancel{2}t}{\cancel{2}}$

$$Y = \frac{\left(\frac{x+5}{2}\right)^2}{3} + 1$$

$$Y = \frac{(x+5)^2}{4 \cdot 3} + 1$$

$$Y = \frac{(x+5)^2}{12} + 1$$

Type 2

use trig identity to help solve...can you recognize the curve

$$x = 3 + 2\sin\theta$$
$$y = 4 - \cos\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 = \sec^2\theta - \tan^2\theta$

hyperbola

$X = 3 + 2\sin\theta$

$\frac{X-3}{2} = \sin\theta$

$Y = 4 - \cos\theta$

$\cos\theta = 4 - Y$

$$\left(\frac{X-3}{2}\right)^2 + (4-Y)^2 = 1$$

$$\frac{(X-3)^2}{4} + (4-Y)^2 = 1$$

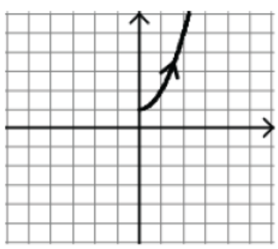
ellipse

Directions: Make a table of values and sketch the curve, indicating the direction of your graph. Then eliminate the parameter.

$$x = \sqrt{t} \qquad y = t + 1$$

Since $x = \sqrt{t}$, we can use only nonnegative values for t.

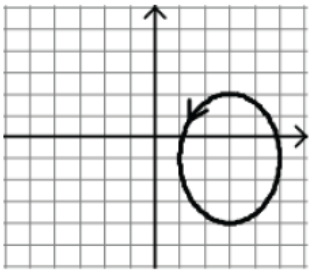
t	0	1	4	9
x	0	1	2	3
y	1	2	5	10



eliminate parameter (let's use x equation)

$x = 3 + 2\cos t \qquad y = -1 + 3\sin t$

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	5	3	1	3	5
y	−1	2	−1	−4	−1



$x = 3 + 2\cos t \qquad y = -1 + 3\sin t$

2. Derivatives and applications

- tangent line /(slope)/ vertical or horizontal
- particle motion (right or left?)
- speed
- 2nd derivative

Example 1

A particle moves in the xy-plane such that its position for time $t \geq 0$ is given by
$$x(t) = 3t^2 - 19t \quad and \quad y(t) = e^{2t-7}$$

What is the slope of the tangent line to the path of the particle when t = 4?

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$x'(t) = 6t - 19$ $\quad$ $y'(t) = e^{2t-7} \cdot (2)$

$$\frac{y'(t)}{x'(t)} = \frac{2e^{2t-7}}{6t-19}$$

$$\frac{y'(4)}{x'(4)} = \frac{2e^{2(4)-7}}{6(4)-19} = \frac{2e}{5}$$

$$\left.\frac{d}{dx}\right|_{t=4}$$

Example 2

Write the equation of the tangent line at t = 2 for the function given parametrically as

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$x(t) = 3t^2 - 4$ $\quad$ $x'(t) = 6t$

$y(t) = \frac{t^3}{3} - t^2 - 3t + 1$

$y'(t) = t^2 - 2t - 3$

$$\frac{dy}{dx} = \frac{t^2 - 2t - 3}{6t}$$

$$\frac{4-4-3}{12} = \frac{-3}{12} = -1/4$$

$\frac{dy}{dx} = -\frac{1}{4}$ $\quad$ $x(2) = 3(2)^2 - 4 = 8$ $\quad$ $y(2) = \frac{8}{3} - 4 - 6 + 1$

$\frac{8}{3} - 9$

$\frac{8}{3} - \frac{27}{3}$

$-\frac{19}{3}$

$(8, -\frac{19}{3})$

$$y + \frac{19}{3} = -\frac{1}{4}(x - 8)$$

- For what value(s) of t would the tangent line to the curve be horizontal?

$\frac{dy}{dx} = \frac{t^2 - 2t - 3}{6t}$ $\quad$ $t^2 - 2t - 3 = 0$

$(t - 3)(t + 1) = 0$

$t = 3, -1$

**instead of asking for t, could ask what's the ordered pair where...

- For what value(s) of t wold the tangent line to the curve be vertical?

$6t = 0$

$t = 0$

Example 3

A particle moves in the xy-plane for t > 0 so that

$$x(t) = t^2 - 4t \quad and \quad y(t) = \ln t$$

- At t = 1, is the particle moving right or left?

$x'(t) = 2t - 4$

$x'(1) = 2(1) - 4$

$= -2$ Left

if have to explain: dx/dt<0 at t=1
don't care about actual number

- Find the speed of the particle at time t = 3.

$$speed = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$x'(t) = 2t - 4$

$y'(t) = \frac{1}{t}$

$\sqrt{(2t-4)^2 + \left(\frac{1}{t}\right)^2}$

$\sqrt{(2(3)-4)^2 + \left(\frac{1}{3}\right)^2}$

$\sqrt{\frac{4}{1} + \frac{1}{9}}$

$\sqrt{\frac{36}{9} + \frac{1}{9}}$

$\sqrt{\frac{37}{9}}$

$\frac{\sqrt{37}}{3}$