AP® CALCULUS BC
2016 SCORING GUIDELINES

Question 6

The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(b) The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

(c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

(d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

Solution	Points
(a) $f(1) = 1,\ f'(1) = -\frac{1}{2},\ f''(1) = \frac{1}{2^2},\ f'''(1) = -\frac{2}{2^3}$ $f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2^2 \cdot 2}(x-1)^2 - \frac{1}{2^3 \cdot 3}(x-1)^3 + \cdots + \frac{(-1)^n}{2^n \cdot n}(x-1)^n + \cdots$	4 : { 1 : first two terms 1 : third term 1 : fourth term 1 : general term
(b) $R = 2$. The series converges on the interval $(-1, 3)$. When $x = -1$, the series is $1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$. Since the harmonic series diverges, this series diverges. When $x = 3$, the series is $1 - 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \cdots$. Since the alternating harmonic series converges, this series converges. Therefore, the interval of convergence is $-1 < x \leq 3$.	2 : { 1 : identifies both endpoints 1 : analysis and interval of convergence
(c) $f(1.2) \approx 1 - \frac{1}{2}(0.2) + \frac{1}{8}(0.2)^2 = 1 - 0.1 + 0.005 = 0.905$	1 : approximation
(d) The series for $f(1.2)$ alternates with terms that decrease in magnitude to 0. $\lvert f(1.2) - T_2(1.2)\rvert \leq \left\lvert \frac{-1}{2^3 \cdot 3}(0.2)^3 \right\rvert = \frac{1}{3000} \leq 0.001$	2 : { 1 : error form 1 : analysis

2016

Name KEY FRQ1: Power Series/Taylor Polynomials

Period:________

2016 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

6. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of f at $x = 1$ is given by $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

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   (c) The Taylor series for f about $x = 1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

   (d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

a) $f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \dots \frac{(-1)^n(n-1)!}{2^n} \frac{(x-c)^n}{n!}$

$1 - \frac{1}{2}(x-1) + \frac{1}{4}\frac{(x-1)^2}{2!} - \frac{1}{4}\frac{(x-1)^3}{3!} + \dots + \frac{(-1)^n(n-1)!}{2^n} \cdot \frac{(x-1)^n}{n!} + \dots$

$f^{(2)}(1) = \frac{(-1)^2(2-1)!}{2^2} = \frac{1}{4}$

$f^{(3)}(1) = \frac{(-1)^3(3-1)!}{2^3}$

$-\frac{1(2)}{8} = -\frac{1}{4}$

b) Radius = 2

interval 1 ± 2

$-1 < x < 3$ check endpoints

$\frac{(-1)^n(n-1)!(x-1)^n}{2^n n!}$

$\frac{(-1)^n(x-1)^n}{2^n n}$

$\frac{(-1)^n(3-1)^n}{2^n n}$

$\frac{(-1)^n 2^n}{2^n n}$ conv. AST

$\frac{(-1)^n(-1-1)^n}{2^n \cdot n}$

$\frac{(-1)^n(-2^n)}{2^n \cdot n} = \frac{(-1)^n(-1)^n 2^n}{2^n n} = \frac{(-1)^{2n}}{n}$ Div. p series

$-1 < x \leq 3$

c) $1 - \frac{1}{2}(1.2-1) + \frac{1}{8}(1.2-1)^2$

$1 - \frac{.2}{2}$ $\frac{.04}{8}$

$f(1.2) \approx 1 - .1 + .005$

$.905$

d) $|f(1.2) - .905| \leq .001$

Alt. series 1st omitted term

$\frac{.2^3}{4 \cdot 3 \cdot 2} = \frac{.008}{4 \cdot 3 \cdot 2}$

$\frac{1}{3000} \leq .001$

Attachments

2005 key.pdf

2005.pdf