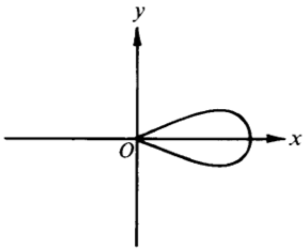
23. 1988



$4\cos(3\theta)=0$ $u=3\theta$

$\cos(3\theta)=0$

$\cos u=0$

$\frac{3\theta}{3}$ $\cancel{u}=\frac{\frac{\pi}{2}+n\pi}{3}$

$\theta=\frac{\pi}{6}+\frac{n\pi}{3}$

Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r=4\cos(3\theta)$ shown in the figure above?

(A) $\cancel{16\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos(3\theta)\,d\theta}$ (B) $\cancel{8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos(3\theta)\,d\theta}$ (C) $\cancel{8\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos^2(3\theta)\,d\theta}$

(D) $16\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$ (E) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$

$\frac{1}{2}\int r^2\,d\theta$

$\frac{1}{2}\int(4\cos(3\theta))^2\,d\theta$

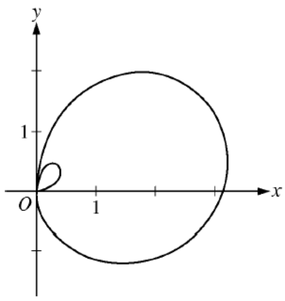
$\frac{1}{2}\int 16\cos^2(3\theta)\,d\theta$

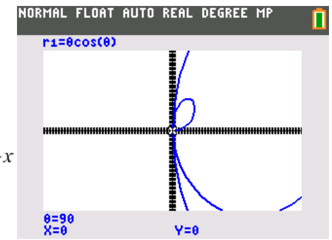
ans

$\theta\cos\theta=0$

$\theta=0$ $\cos\theta=0$

$\theta=\frac{\pi}{2}$





2. Consider the polar curve defined by the function $r(\theta)=\theta\cos\theta$, where $0\le\theta\le\frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta}=\cos\theta-\theta\sin\theta$. The figure above shows the graph of r for $0\le\theta\le\frac{3\pi}{2}$.

(a) Find the area of the region enclosed by the inner loop of the curve.

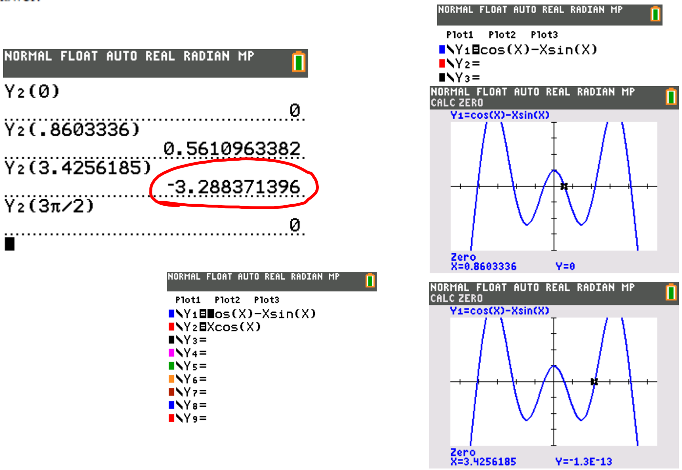
$\frac{1}{2}\int r^2\,d\theta$

$\frac{1}{2}\int_0^{\pi/2}(\theta\cos\theta)^2\,d\theta$

(a) Area $=\frac{1}{2}\int_0^{\pi/2}(r(\theta))^2\,d\theta=0.127$ (or 0.126)	3 : { 1 : integrand 1 : limits and constant 1 : answer

2. Consider the polar curve defined by the function $r(\theta) = \theta\cos\theta$, where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$.

(b) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.



(b) $r'(\theta) = \cos\theta - \theta\sin\theta = 0 \Rightarrow \theta = 0.860334,\ 3.425618$

θ	$r(\theta)$
0	0
0.860334	0.561096
3.425618	−3.288371
$\frac{3\pi}{2}$	0

Therefore, the greatest distance from any point on the graph of r to the origin is 3.288.

$3 : \begin{cases} 1 : \text{identifies } \theta = 3.425618 \text{ as a candidate} \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

$\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{2}{2-\pi} = \frac{\frac{1}{2}}{\frac{dx}{d\theta}}$$

or $\frac{2}{2-\pi} = \frac{\frac{1}{2}}{x'}$

$$\frac{2x'}{2} = \frac{(2-\pi)\frac{1}{2}}{2}$$

$$x' = \frac{(2-\pi)}{4}$$

(c) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

At the point where the tangent line has slope $\frac{2}{2-\pi}$,

$\frac{2}{2-\pi} = \frac{1/2}{dx/d\theta}$.

Therefore, $\frac{dx}{d\theta} = \frac{1}{2} \cdot \frac{2-\pi}{2} = \frac{2-\pi}{4}$ at this point.

$3 : \begin{cases} 1 : \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$