Area of Polar Region

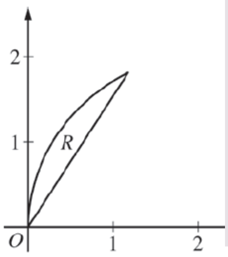
Typical AP question:
Which one of these integrals represents the area?

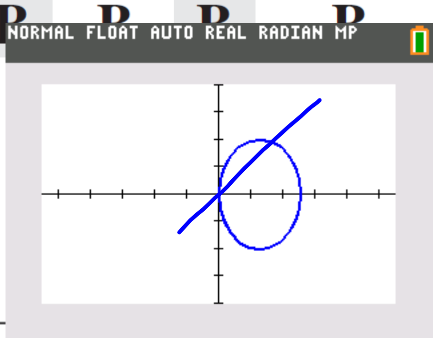
B B B B B

$\frac{4\cos\theta}{4} = \frac{0}{4}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2} + n\pi$



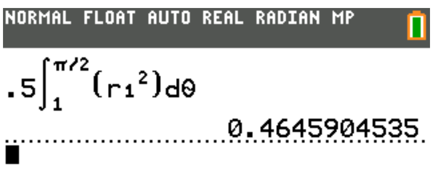


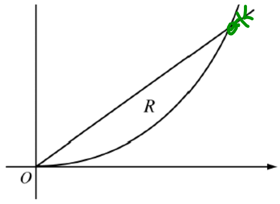
88. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R ?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819

$\frac{1}{2}\int r^2 \, d\theta$

$\frac{1}{2}\int_1^{\pi/2} (4\cos\theta)^2 \, d\theta$





16. Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and $\theta = k$, where k is a constant, $0 < k < \frac{\pi}{2}$, as shown in the figure above. What is the area of R in terms of k ?

(A) $\frac{k^3}{6}$ (B) $\frac{k^3}{3}$ (C) $\frac{k^3}{2}$ (D) $\frac{k^2}{4}$ (E) $\frac{k^2}{2}$

ans

$\frac{1}{2}\int_0$

$\frac{1}{2}\int_0$

$$\frac{1}{2}\int_a^b r^2\, d\theta$$

$$\frac{1}{2}\int_0^k \theta^2\, d\theta$$

$$\frac{1}{2}\left[\frac{\theta^3}{3}\right]_0^k$$

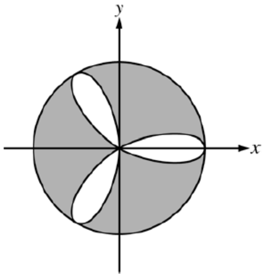
$F(k) - F(0)$

$$\frac{1}{2}\left[\frac{k^3}{3} - 0\right]$$

$$k^3/6$$

Area Between Two Polar Curves

$$A = \frac{1}{2}\int_\alpha^\beta \left(\left(r_{outer}\right)^2 - \left(r_{inner}\right)^2\right) d\theta$$



Area of Circle − Area of Rose curve

πr^2 −

$\pi(2)^2$ −

4π −

91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

answer...D

$\frac{1}{2}\int_a^b r^2\, d\theta$

one petal

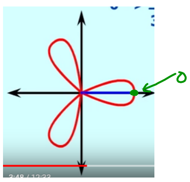
$2\left(\frac{1}{2}\int_0^{\pi/6}(2\cos 3\theta)^2\, d\theta\right)$

1.047×3 petals ≈ 3.14159

$4\pi - 3.14159 \approx 9.425$

Given $r = 2\cos 3\theta$ (no calculator)

determine area of one leaf.



$u=3\theta$

$2\cos 3\theta = 0$

$\cos 3\theta = 0$

$\cos u = 0$

$\frac{3\theta}{3} = \frac{\frac{\pi}{2}+n\pi}{3}$

$\theta = \frac{\pi}{6} + \frac{n\pi}{3}$

$2\left(\frac{1}{2}\int_0^{\pi/6}(2\cos 3\theta)^2\, d\theta\right)$

one petal

$\int_0^{\pi/6} 4\cos^2(3\theta)\, d\theta$

$4\int_0^{\pi/6} \cos^2(3\theta)\, d\theta$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\cos^2 3\theta = \frac{1}{2}(1+\cos 6\theta)$

$4 \cdot \frac{1}{2}\int_0^{\pi/6} 1+\cos 6\theta\, d\theta$

$\int \cos 6\theta\, d\theta$

$u = 6\theta$

$du = 6\,d\theta$

$\frac{1}{6}du = d\theta$

$2\int_0^{\pi/6} 1+\cos 6\theta\, d\theta$

$2\left[\theta + \frac{1}{6}\sin 6\theta\right]_0^{\pi/6}$

$\frac{1}{6}\int \cos u\, du$

$\frac{1}{6}\sin 6\theta$

$F(\pi/6) - F(0)$

$2\left[\frac{\pi}{6} + \frac{1}{6}\cancel{\sin\pi}^{\,0} - 0\right]$

$\frac{2\pi}{6}$

$\frac{\pi}{3}$ one petal