2. Derivatives

Find the slope of the curve

$r = 3\sin 2\theta \; at \; \theta = \frac{\pi}{3}$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$x = r\cos\theta$

$y = r\sin\theta$

$x = r\cos\theta$

$x = 3\sin 2\theta \cos\theta$ product rule

$\frac{dx}{d\theta} = 3\sin 2\theta(-\sin\theta) + \cos\theta(3\cos 2\theta)(2)$

$\frac{dx}{d\theta} = -3\sin 2\theta \sin\theta + 6\cos\theta\cos 2\theta$

don't spend a lot of time simplifying but if obvious pyth. id. or like terms combine

$y = r\sin\theta$

$y = 3\sin 2\theta \sin\theta$

$\frac{dy}{d\theta} = 3\sin 2\theta(\cos\theta) + \sin\theta(3\cos 2\theta)(2)$

$\frac{dy}{d\theta} = 3\sin 2\theta\cos\theta + 6\sin\theta\cos 2\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\sin 2\theta\cos\theta + 6\sin\theta\cos 2\theta}{-3\sin 2\theta\sin\theta + 6\cos\theta\cos 2\theta}$

$\frac{y'(\pi/3)}{x'(\pi/3)} = \frac{3\sin(\frac{2\pi}{3})\cos\frac{\pi}{3} + 6\sin\frac{\pi}{3}\cos\frac{2\pi}{3}}{-3\sin(\frac{2\pi}{3})\sin\frac{\pi}{3} + 6\cos\frac{\pi}{3}\cos\frac{2\pi}{3}}$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2})\; (\frac{1}{2}, \frac{\sqrt{3}}{2})$

$= \frac{3(\frac{\sqrt{3}}{2})(\frac{1}{2}) + 6(\frac{\sqrt{3}}{2})(-\frac{1}{2})}{-3(\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2}) + 6(\frac{1}{2})(-\frac{1}{2})}$

$\frac{\frac{3\sqrt{3}}{4} - \frac{6\sqrt{3}}{4}}{-\frac{9}{4} - \frac{6}{4}} = \frac{\frac{-3\sqrt{3}}{4}}{\frac{-15}{4}} = \frac{-3\sqrt{3}}{-15} = \frac{\sqrt{3}}{5}$

2. Derivatives

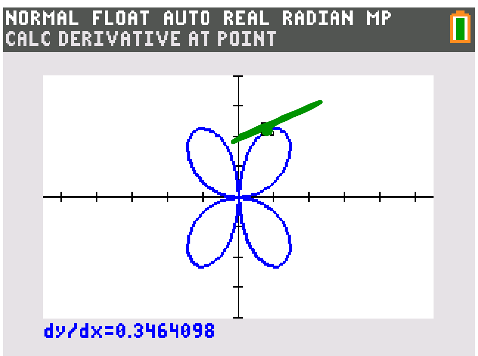
Find the slope of the curve

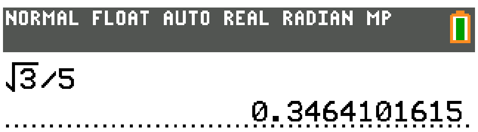
$$r = 3\sin 2\theta \; at \; \theta = \frac{\pi}{3}$$

verify with calculator...although most times on non-calculator

mode: polar

2nd calc dy/dx





26. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

answer....B

$x = r\cos\theta$

$x = (1+2\sin\theta)\cos\theta$

$x' = (1+2\sin\theta)(-\sin\theta) + \cos\theta(2\cos\theta)$

$x' = -\sin\theta(1+2\sin\theta) + 2\cos^2\theta$

$x'(0) = 0(1) + 2(1)^2$

$x'(0) = 2$

$y = r\sin\theta$

$y = (1+2\sin\theta)\sin\theta$

$y' = (1+2\sin\theta)(\cos\theta) + \sin\theta(2\cos\theta)$

$y'(0) = 1(1) + 0(2)$

$y'(0) = 1$

$\frac{y'}{x'} = \frac{1}{2}$

25. What is the slope of the line tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{6}$?

(A) $-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

an

$x = r\cos\theta$

$x = \cos\theta\cos\theta$

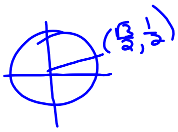
$x = (\cos\theta)^2$

$x' = 2(\cos\theta)^1(-\sin\theta)$

$x' = -2\cos\theta\sin\theta$

$x' = -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$x' = -\frac{\sqrt{3}}{2}$



$y = r\sin\theta$

$y = \cos\theta\sin\theta$

$y' = \cos\theta(\cos\theta) + \sin\theta(-\sin\theta)$

$y' = \cos^2\theta - \sin^2\theta$ → $\cos 2\theta$

$y' = (\cos\frac{\pi}{6})^2 - (\sin\frac{\pi}{6})^2$ $\quad \cos 2\left(\frac{\pi}{6}\right)$

$y' = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$ $\quad \cos\left(\frac{\pi}{3}\right)$

$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ $\quad \frac{1}{2}$

$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$

3. Integrals

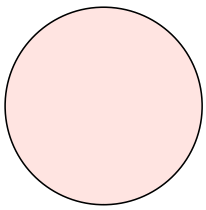
Area Enclosed by Polar Curves

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta$$

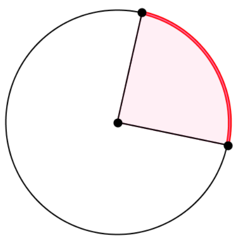
helpful formulas: if integrating by hand

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $= \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $= \frac{1+\cos 2\theta}{2}$



$$A = \pi r^2$$



$$A = \frac{\theta}{2\pi}\left(\pi r^2\right)$$

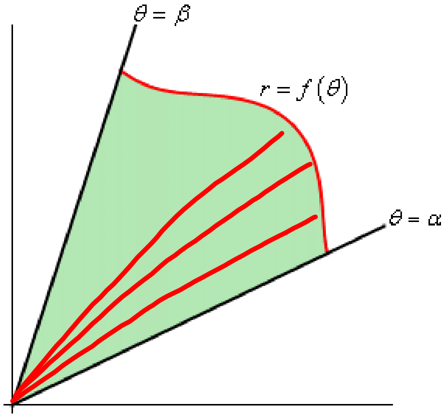
sector

ex. show half and quarter circle

$$\frac{\pi/2}{2\pi} = \frac{\pi}{4\pi} = \frac{1}{4}$$

$$A = \frac{\theta}{2} r^2$$

$$A = \frac{1}{2} r^2 \theta$$



$$A = \frac{1}{2} r^2 \theta$$

polar shape

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 \, d\theta$$

Area of Polar Region

Typical AP question:
Which one of these integrals represents the area?