Example 3

For $t \geq 0$, a particle is moving along a curve so that its position at time t is (x(t), y(t)).

At time t =1, that particle is at a position (2, 7). It is known that

when t=1, x=2, y=7

$$\underset{x'(t)}{\frac{dx}{dt}} = \frac{2}{t^2+3} \quad and \quad \underset{y'(t)}{\frac{dy}{dt}} = \cos^3 t$$

- Find the x-coordinate of the particle's position at time t = 4.

$$\int_a^b f(x) = F(b) - F(a)$$

$$\int_1^4 x'(t)\,dt =$$

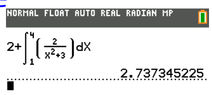
$$x(t)\Big]_1^4 = F(4) - F(1)$$

$$(x(t))\Big]_1^4 = F(4) - 2$$

$$\int_1^4 \frac{2}{t^2+3}\,dt = F(4) - 2$$

$$2 + \int_1^4 \frac{2}{t^2+3}\,dt = F(4)$$

2.737



- Find the y-coordinate of the particle's position at time t = 4.

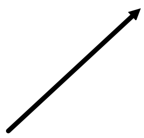
$$\frac{dy}{dt} = \cos^3 t$$

$$\int_1^4 \cos^3 t + 7 =$$

5.745

Vectors

have direction and magnitude



go hand in hand with parametric equations

Vectors:

1. Particle Motion / Speed
2. Acceleration, Speed and Position
3. Tangent Line

Remember different types of notation

position: $s(t)$	$\langle x(t), y(t) \rangle$	
velocity: $v(t)$	$\langle x'(t), y'(t) \rangle$ $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$	} all ways of stating first derivative
acceleration: $a(t)$	$\langle x''(t), y''(t) \rangle$	

Example 1

The movement of a particle in the xy-plane is given by

$$(x(t), y(t)) \text{ where } x(t) = e^t + 1 \text{ and } y(t) = t^2 + 2$$

$x'(t) = e^t$ $y'(t) = 2t$

What is the speed of the particle at time t = 2?

$\sqrt{(x'(t))^2 + (y'(t))^2}$

$\sqrt{(e^t)^2 + (2t)^2}$ when $t=2$

≈ 8.402

Example 2

The position of a particle moving in the xy-plane is determined by the curve

$$(x(t), y(t)), x'(t) = 3e^{3t}, \text{ and } y'(t) = 2t - 4$$

At t=0, the particle is at the point (0,0)

a. Compute the speed and the acceleration vector of the particle at t=0.

Speed: $\sqrt{(3e^{3t})^2 + (2t-4)^2}$

$\sqrt{9+16} = \sqrt{25} = 5$

acceleration $\langle 3e^{3t}, 2t-4 \rangle$

$\langle 3e^{3t} \cdot 3, 2 \rangle$

$\langle 9e^{3t}, 2 \rangle$

when $t=0$ $\langle 9, 2 \rangle$

b. Compute the x and y coordinate of the particle at t=4. $t=0$ $(0,0)$ (x,y)

$\int_0^4 3e^{3t}\,dt + 0$ $\int_0^4 2t-4\,dt + 0$

Example 3

Given: $x(t) = 2t^2 - 3t$

$y(t) = e^{3t-6}$

$x(2) = 2(2)^2 - 3(2) = 2$

$y(2) = e^{6-6} = 1$

$(2, 1)$

- Write the equation of the line tangent to the particle's path at t = 2.

$\frac{y'}{x'} = \frac{3e^{3t-6}}{4t-3}$ when $t=2$ $\frac{3}{5}$

$y - 1 = \frac{3}{5}(x - 2)$

- When would the path have a vertical tangent?

$4t - 3 = 0$

$t = 3/4$ as long as $y' \neq 0$

Speed = $|v(t)|$

Velocity is a vector quantity; that is, it has both a direction and a magnitude. The magnitude of velocity vector is the speed. Speed is a non-negative number and has no direction associated with it.Velocity has a magnitude and a direction. Speed has the same value and units as velocity; speed is a number.

Common AP questions:

When is speed **increasing** or when is speed **decreasing**?

Speed is increasing when both the velocity and the acceleration have the same signs. Think of an object which is thrown into the air. It starts with a positive velocity, so it is rising. But, its speed is decreasing because its acceleration (gravity) is negative. When it reaches its maximum height, the velocity is temporarily 0, and then becomes negative as the particle starts to fall back to the ground. Now the acceleration and the velocity are both negative, which is why the speed of the particle is increasing.

Picture a car moving along a road going forwards (in the positive direction) its velocity is positive.

> If you step on the gas your, acceleration pulls you in the direction you are moving and your speed increases. (v > 0, a > 0, speed increases)

> Going too fast is not good, so you put on your breaks, you now accelerate in the opposite direction (decelerate?), but you are still moving forward, but slower. (v > 0, a < 0, speed decreases)

> Finally you stop. Then you shift into reverse and start moving backwards (negative velocity) and you push on the gas to accelerate in the negative direction, so your speed increases. (v < 0, a < 0, speed increases)

> Then you put on the breaks (accelerate in the positive direction) and your speed decreases again. (v < 0, a > 0, speed decreases)

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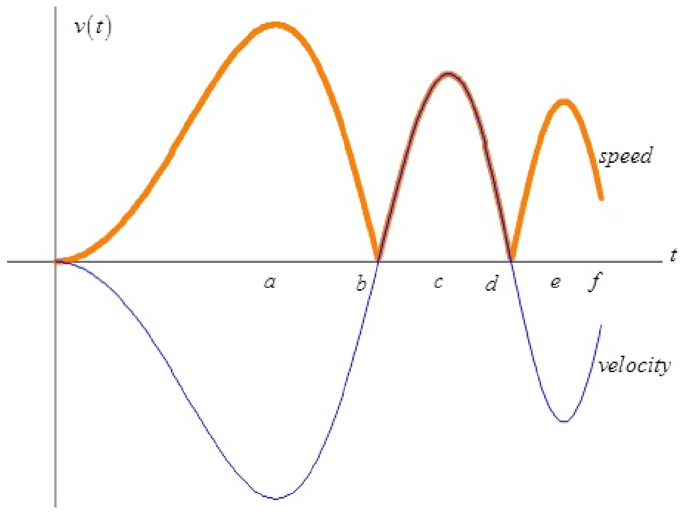
The figure below shows the graph on velocity v(t) (blue graph) of a particle moving on the interval $0 \le t \le f$. The red graph is $|v(t)|$ the speed. The sections where v(t) < 0 are reflected over the x-axis. The graphs overlap on [b, d]. It is now quite easy to see that the speed is increasing on the intervals [0, a], [b, c] and [d, e].



Common AP questions:

When is speed **increasing** or when is speed **decreasing**?

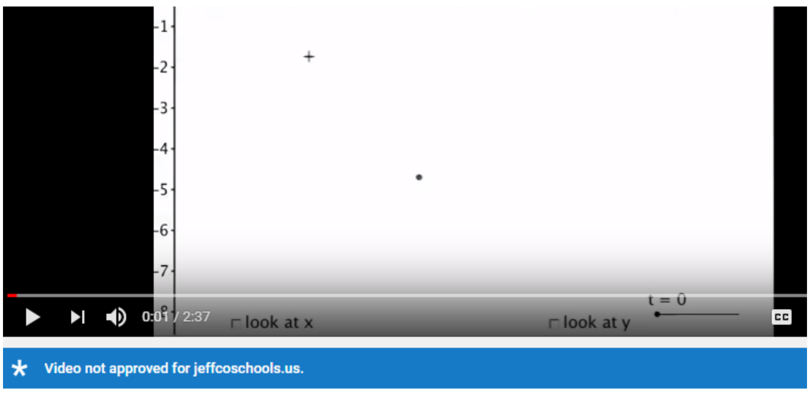
Speed is increasing when both the velocity and the acceleration have the same signs.

both positive or both negative

parametric

what does dx/dt and dy/dt look like

https://www.youtube.com/watch?v=3bXUIQY8p54



Calc BC Parametric Derivatives 1