AP® CALCULUS BC
2012 SCORING GUIDELINES

Question 6

The function g has derivatives of all orders, and the Maclaurin series for g is $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots$.

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

(a) $\left|\frac{x^{2n+3}}{2n+5} \cdot \frac{2n+3}{x^{2n+1}}\right| = \left(\frac{2n+3}{2n+5}\right) \cdot x^2$

$\lim_{n\to\infty}\left(\frac{2n+3}{2n+5}\right) \cdot x^2 = x^2$

$x^2 < 1 \Rightarrow -1 < x < 1$
The series converges when $-1 < x < 1$.

When $x = -1$, the series is $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$
This series converges by the Alternating Series Test.

When $x = 1$, the series is $\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \cdots$
This series converges by the Alternating Series Test.

Therefore, the interval of convergence is $-1 \le x \le 1$.

5 : { 1 : sets up ratio; 1 : computes limit of ratio; 1 : identifies interior of interval of convergence; 1 : considers both endpoints; 1 : analysis and interval of convergence }

(b) $\left|g\left(\frac{1}{2}\right) - \frac{17}{120}\right| < \frac{\left(\frac{1}{2}\right)^5}{7} = \frac{1}{224} < \frac{1}{200}$

2 : { 1 : uses the third term as an error bound; 1 : error bound }

(c) $g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \cdots + (-1)^n\left(\frac{2n+1}{2n+3}\right)x^{2n} + \cdots$

2 : { 1 : first three terms; 1 : general term }

worked

2012 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

check endpoints

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

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a) $\lim_{n\to\infty} \left| \dfrac{\frac{(-1)^{n+1} x^{(2(n+1)+1}}{2(n+1)+3}}{\frac{(-1)^n x^{2n+1}}{2n+3}} \right|$

$\lim_{n\to\infty} \frac{x^{2n+3}}{2n+5} \cdot \frac{2n+3}{x^{2n+1}}$

$\lim_{n\to\infty} \frac{x^{2n+3}}{x^{2n+1}} \cdot \frac{2n+3}{2n+5}$

$x^2 < 1$

$-1 < x < 1$ check end points

$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{2n+1}}{2n+3}$ $\sum_{n=0}^{\infty} \frac{(-1)^n(1)^{2n+1}}{2n+3}$

$\sum_{n=0}^{\infty} \frac{(-1)^{3n+1}}{2n+3}$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+3}$

conv. by AST conv. By AST

$-1 \le x \le 1$

$[-1, 1]$

b) because alternating estimate will be less than 1st omitted term

$\frac{x^5}{7}$ $\frac{(\frac{1}{2})^5}{7}$

$\frac{1}{32 \cdot 7}$

$\frac{32 \cdot 7}{224}$

$\frac{1}{224} < \frac{1}{200}$

STOP

END OF EXAM

c) $g'(x) = \frac{1}{3} - \frac{3x^2}{5} + \frac{5x^4}{7} + \ldots + \frac{(-1)^n(2n+1)x^{2n}}{2n+3} + \ldots$

Attachments

2005 key.pdf

2005.pdf