## AP® CALCULUS BC 2012 SCORING GUIDELINES

## Question 6

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at  $x = \frac{1}{2}$  is an alternating series whose terms decrease in absolute value to 0. The approximation for  $g(\frac{1}{2})$  using the first two nonzero terms of this series is  $\frac{17}{120}$ . Show that this approximation differs from  $g(\frac{1}{2})$  by less than  $\frac{1}{200}$ .

(c) Write the first three nonzero terms and the general term of the Maclaurin series for g'(x).

(a) 
$$\left| \frac{x^{2n+3}}{2n+5} \cdot \frac{2n+3}{x^{2n+1}} \right| = \left( \frac{2n+3}{2n+5} \right) \cdot x^2$$

$$\lim_{n\to\infty} \left(\frac{2n+3}{2n+5}\right) \cdot x^2 = x^2$$

$$x^2 < 1 \implies -1 < x < 1$$

The series converges when -1 < x < 1.

When x = -1, the series is  $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$ 

This series converges by the Alternating Series Test

When x = 1, the series is  $\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \cdots$ 

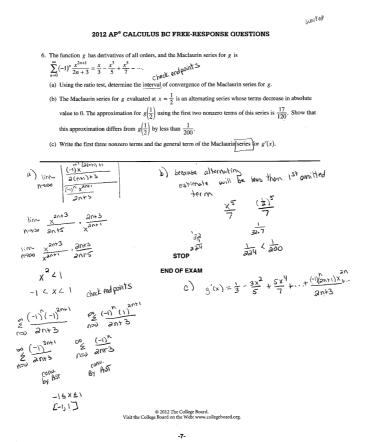
This series converges by the Alternating Series Test.

Therefore, the interval of convergence is  $-1 \le x \le 1$ .

(b) 
$$\left| g\left(\frac{1}{2}\right) - \frac{17}{120} \right| < \frac{\left(\frac{1}{2}\right)^5}{7} = \frac{1}{224} < \frac{1}{200}$$

(c) 
$$g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \dots + (-1)^n \left(\frac{2n+1}{2n+3}\right)x^{2n} + \dots$$
 2 :  $\begin{cases} 1 : \text{ first three terms} \\ 1 : \text{ general term} \end{cases}$ 

 $2: \left\{ \begin{array}{l} 1: uses \ the \ third \ term \ as \ an \ error \ bound \\ 1: error \ bound \end{array} \right.$ 



2005 key.pdf

2005.pdf