p - series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

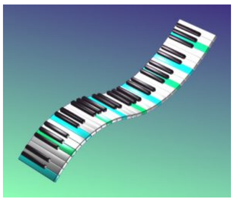
if $p > 1$ converges

if $0 < p \leq 1$ diverges

when p=1 special name "harmonic series" ***diverges***

read p. 619 grey box

The name originated with the Greeks, who as we know had words for many things. It was Pythagoras who was the first person to study the notes emitted by plucked strings of various lengths. If a string which emits middle C when plucked is reduced to two-thirds of its length, it will emit the note G (musicians call the interval from C to G a *fifth*). If the string is halved in length, it will emit top C, an octave higher. These notes are fundamental to the Pythagorean theory of harmony, and the corresponding lengths of string.



$$\sum_{n=1}^{\infty} \frac{1}{n}$$

DIVERGES

The sequence converges, but the series diverges.

Ex. Use p-series to determine whether the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$ $\quad 5/3 > 1$ conv. by p series

b) $\sum_{n=1}^{\infty} \frac{2}{\sqrt[5]{n^2}} = \frac{2}{n^{2/5}}$ $\quad 2/5 \leq 1$ diverges by p series

c) $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \ldots$

$n=1 \quad n=2 \quad n=3 \quad n=4$

$\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ $\quad 2/3 \leq 1$ div. by p-series

d) $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \ldots$

$n=1 \quad n=2 \quad n=3 \quad n=4$

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ $\quad 3 > 1$ conv. by p series

In summary...are you going to confuse

r in a geometric series $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

$$|r| < 1 \quad converges$$

p in a p -series $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$$p > 1 \quad converges$$