9.9 Representation of Functions by Power Series (Geometric)

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n, \quad |x|<1$$

write out the first few terms...

$$1+x+x^2+x^3+x^4+\ldots$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

~ to Geometric series

we want in the form $\dfrac{1}{1-(something)}$

Ex. 1 $\sum$ has x's

Find a power series
and interval of convergence
centered at 0
for the function

$$\boxed{\frac{1}{1-(something)}}$$

$$f(x) = \frac{1}{1+x}$$

$$\frac{1}{1-(-x)}$$

$$\sum_{n=0}^{\infty} (-x)^n$$

$$\sum_{n=0}^{\infty} (-1)^n (x)^n$$

Ex. 2
Find a power series
and interval of convergence
centered at 0
for the function

$$\boxed{\frac{1}{1-(something)}}$$

$$f(x)=\frac{1}{1-(x^3)}$$

$$\sum_{n=0}^{\infty}(x^3)^n$$

$$\sum_{n=0}^{\infty}x^{3n}$$

Ex. 3
Find a power series
and interval of convergence
centered at 0
for the function

$$\boxed{\frac{1}{1-(something)}}$$

$$f(x)=\frac{1}{1+9x^2}$$

$$\frac{1}{1-(-9x^2)}$$

$$\sum_{n=0}^{\infty}(-9x^2)^n$$

$$\sum_{n=0}^{\infty}(-1)^n(9)^n x^{2n}$$

Ex. 4

Find a power series centered at 0 for the function

$$\boxed{\frac{1}{1-(something)}}$$

$$f(x)=\frac{x}{4x+1}$$

$$f(x)=x\left(\frac{1}{4x+1}\right)$$

$$f(x)=x\left(\frac{1}{1+4x}\right)$$

$$f(x)=x\left(\frac{1}{1-(-4x)}\right)$$

$$x\sum_{n=0}^{\infty}(-4x)^n$$

$$x\sum_{n=0}^{\infty}(-1)^n 4^n x^n$$

$$\sum_{n=0}^{\infty}(-1)^n 4^n x^{n+1}$$

Ex. 5

Find a power series centered at 0 for the function

$$\boxed{\frac{1}{1-(something)}}$$

$$f(x)=\frac{x}{9+x^2}$$

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Power Series for Elementary Functions

POWER SERIES FOR ELEMENTARY FUNCTIONS

Function	*Interval of Convergence*
$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - \cdots + (-1)^n(x-1)^n + \cdots$	$0 < x < 2$
$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots + (-1)^n x^n + \cdots$	$-1 < x < 1$
$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots + \frac{(-1)^{n-1}(x-1)^n}{n} + \cdots$	$0 < x \leq 2$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \cdots$	$-\infty < x < \infty$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$	$-\infty < x < \infty$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$	$-\infty < x < \infty$
$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$	$-1 \leq x \leq 1$
$\arcsin x = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \cdots + \frac{(2n)!x^{2n+1}}{(2^n n!)^2(2n+1)} + \cdots$	$-1 \leq x \leq 1$
$(1+x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \frac{k(k-1)(k-2)(k-3)x^4}{4!} + \cdots$	$-1 < x < 1$*

* *The convergence at $x = \pm 1$ depends on the value of k.*