Intervals of Convergence

$f(x), \quad f'(x), \quad \int f(x)dx$

$$f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}=x+\frac{x^2}{2}+\frac{x^3}{3}...$$

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{n}{n+1}\cdot\frac{x^n x^1}{x^n}\right|$$

1

$|x|<1$

$-1<x<1$ Radius: 1

$\sum_{n=1}^{\infty}\frac{x^n}{n}$ $\sum_{n=1}^{\infty}\frac{x^n}{n}$

$\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ Conv. AST

$\sum_{n=1}^{\infty}\frac{1^n}{n}$ Div. p-series

$-1\le x<1$

$[-1,1)$

Intervals of Convergence

$f'(x)$ $(-1,1)$ $-1<x<1$

$$f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}=x+\frac{x^2}{2}+\frac{x^3}{3}...$$

$$\sum_{n=1}^{\infty}\frac{\cancel{n}x^{n-1}}{\cancel{n}}=1+\frac{\cancel{2}x^1}{\cancel{2}}+\frac{\cancel{3}x^2}{\cancel{3}}$$

$$\sum_{n=1}^{\infty}x^{n-1}=1+x+x^2$$

$\sum_{n=1}^{\infty}(-1)^{n-1}$ $\sum_{n=1}^{\infty}(1)^{n-1}$

Both Div.

$(-1,1)$

$-1<x<1$

Intervals of Convergence

$$\int f(x)dx$$

$$f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}=x+\frac{x^2}{2}+\frac{x^3}{3}\ldots$$

$$\sum_{n=1}^{\infty}\frac{x^{n+1}}{(n+1)n}=\frac{x^2}{2}+\frac{x^3}{3\cdot 2}+\frac{x^4}{4\cdot 3}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)}$$

conv. by AST

$$\sum_{n=1}^{\infty}\frac{1^{n+1}}{n(n+1)}=\frac{1}{n^2+n}$$

LCT to $\frac{1}{n^2}$

conv.

$-1\leq x\leq 1$

$[-1,1]$

Summary

- A power series representation of a function $f(x)$ can be differentiated term-by-term to obtain a power series representation of its derivative $f'(x)$. The interval of convergence of the differentiated series is the same as that of the original series.
- A power series representation of a function $f(x)$ can be anti-differentiated term-by-term to obtain a power series representation of its anti-derivative $\int f(x)\,dx$. The value of the constant of integration, C, can be determined by substituting the center of the power series for x. The interval of convergence of the anti-differentiated series is the same as that of the original series.
- A power series representation of a function $f(x)$ can be integrated term-by-term from a to b to obtain a series representation of the definite integral $\int_a^b f(x)\,dx$, provided that the interval (a,b) lies within the interval of convergence of the power series that represents $f(x)$.