Ex. 2

$$\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$$

$$\lim_{n\to\infty} \left| \frac{(3x)^{n+1}}{(2(n+1))!} \cdot \frac{(2n)!}{(3x)^n} \right|$$

$(2n+2)!$

$$\lim_{n\to\infty} \left| \frac{\cancel{(3x)^n}(3x)^1}{\cancel{(3x)^n}} \cdot \frac{\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{3x}{(2n+2)(2n+1)} \right|$$

$0 < 1$ conv.

Domain $(-\infty, \infty)$

Radius ∞

Ex. 3

$$\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)4^{n+1}}$$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1+1}}{(n+1+1)4^{n+1+1}} \cdot \frac{(n+1)4^{n+1}}{(x-2)^{n+1}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+2}}{(x-2)^{n+1}} \cdot \frac{(n+1)}{(n+2)} \cdot \frac{4^{n+1}}{4^{n+2}} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x-2)^n}(x-2)^{\cancel{2}1}}{\cancel{(x-2)^n}\cancel{(x-2)^1}} \cdot \frac{n+1}{n+2} \cdot \frac{\cancel{4^n}\cdot\cancel{4^1}}{\cancel{4^n}\cdot 4^{\cancel{2}1}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-2)}{4} \cdot \cancel{\frac{n+1}{n+2}} \right|$$

1

$$\left| \frac{x-2}{4} \right| < 1$$

$$-1 < \frac{x-2}{4} < 1$$

$$-4 < x-2 < 4$$

$+2 \quad +2 \quad +2$

$$-2 < x < 6$$

Radius $= 4$

$$\sum_{n=0}^{\infty} \frac{(\cancel{x}^{-2}-2)^{n+1}}{(n+1)4^{n+1}} \qquad \sum_{n=0}^{\infty} \frac{(\cancel{x}^{6}-2)^{n+1}}{(n+1)4^{n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-2-2)^{n+1}}{(n+1)4^{n+1}} \qquad \sum_{n=0}^{\infty} \frac{(6-2)^{n+1}}{(n+1)4^{n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-4)^{n+1}}{(n+1)4^{n+1}} \qquad \sum_{n=0}^{\infty} \frac{\cancel{4^{n+1}}}{(n+1)\cancel{4^{n+1}}}$$

$$\sum_{n=0}^{\infty} \frac{(-1\cdot 4)^{n+1}}{(n+1)4^{n+1}} \qquad \sum_{n=0}^{\infty} \frac{1}{n+1}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\cancel{4^{n+1}}}{(n+1)\cancel{4^{n+1}}}$$

LCT to $\sum \frac{1}{n}$ diverges

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

conv. AST

$$-2 \le x < 6$$

$[-2, 6)$ interval of conv.

Ex. 4

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n2^n}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x-2)^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n2^n}{(-1)^{n+1}(x-2)^n}\right|$$

$$\lim_{n\to\infty}\frac{\cancel{(x-2)^n}(x-2)^1\cdot n\cdot\cancel{2^n}}{\cancel{(x-2)^n}\cdot n+1\cdot\cancel{2^n}\cdot 2^1}$$

$$\lim_{n\to\infty}\frac{x-2}{2}\cdot\frac{\cancel{n}}{\cancel{n}+1}$$

$$1$$

$$\left|\frac{x-2}{2}\right|<1$$

$$-1<\frac{x-2}{2}<1$$

$$-2<x-2<2$$
$$+2 \quad +2\ +2$$

$$0<x<4$$

Radius: 2

Test end points
$0<x<4$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(\overset{0}{x}-2)^n}{n2^n} \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(\overset{4}{x}-2)^n}{n2^n}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-2)^n}{n\,2^n} \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cancel{(2)^n}}{n\cancel{2^n}}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n\cdot\cancel{2^n}}{n\,\cancel{2^n}} \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$$

conv. by AST

$$\sum_{n=1}^{\infty}\frac{(-1)^{2n+1}}{n}$$

$$\sum_{n=1}^{\infty}\frac{-1}{n}$$

p-series $\frac{1}{n}$ div.

$$0<x\le 4$$

$$(0,4]$$

interval of conv.

2 more examples if need it

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$$

2 more examples if need it

$$\sum_{n=0}^{\infty} n^3 (x-5)^n$$

Ex.

$$f(x)=\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

$|r|<1$

$\left|\frac{x}{2}\right|<1$

$-1<\frac{x}{2}<1$

$-2<x<2$

Geometric will <u>NEVER</u> include endpoints.

<u>Derivatives/Integrals of Power Series</u>

$$\boxed{f(x)=\sum_{n=0}^{\infty}a_n(x-c)^n}$$

Be careful u-sub.

$$f'(x)=\sum_{n=1}^{\infty}na_n(x-c)^{n-1}$$

$$\int f(x)=\sum_{n=0}^{\infty}\frac{a_n(x-c)^{n+1}}{n+1}+c$$

The *radius of convergence* of the series obtained by differentiating or integrating a power series is the **SAME** as that of the ORIGINAL power series. The *interval of convergence*, however **may differ** as a result of the behavior at the **endpoints**.

Find the interval of convergence of f'(x).
Be sure to check endpoints.

$-2<x<2$

$$f(x)=\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

$$f'(x)=\sum_{n=0}^{\infty} n\left(\frac{x}{2}\right)^{n-1}\cdot\frac{1}{2}$$

$$\sum_{n=0}^{\infty}\frac{n}{2}\left(\frac{x}{2}\right)^{n-1}$$

endpoints

-2

$$\sum_{n=0}^{\infty}\frac{n}{2}\left(\frac{-2}{2}\right)^{n-1}$$

$$\sum_{n=0}^{\infty}\frac{n}{2}(-1)^{n-1}$$

div.

2

$$\sum_{n=0}^{\infty}\frac{n}{2}\left(\frac{2}{2}\right)^{n-1}$$

$$\sum_{n=0}^{\infty}\frac{n}{2}(1)^{n-1}$$

div.

$-2<x<2$

Find the interval of convergence of ∫f(x).

Be sure to check endpoints.

$-2<x<2$

$$f(x)=\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

$u=\frac{x}{2}$

$du=\frac{1}{2}dx$

$2du=dx$

$$\int f(x)=$$

$$\sum_{n=0}^{\infty}\frac{2\left(\frac{x}{2}\right)^{n+1}}{n+1}$$

endpoints

$$\sum_{n=0}^{\infty}\frac{2\left(\frac{-2}{2}\right)^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty}\frac{2(-1)^{n+1}}{n+1}$$

Conv. by AST

$-2\le x<2$

$$\sum_{n=0}^{\infty}\frac{2\left(\frac{2}{2}\right)^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty}\frac{2(1)^{n+1}}{n+1}$$

div.