Ex. 4

(whole question)

The function f has derivatives of all orders for all real numbers x. Assume that

$$f(2)=6,\ f'(2)=4,\ f''(2)=-7,\ f'''(2)=8$$

(a) Write the third-degree Taylor polynomial for f about $x = 2$, and use it to approximate $f(2.3)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 9$ for all x in the closed interval [2, 2.3]. Use the Lagrange error bound on the the apporximation of f (2.3) found in part (a) to find an interval [a, b] such that

(c) Could $f(2.3)$ equal 6.922? Show why or why not.

Ex. 4

The function f has derivatives of all orders for all reall numbers x. Assume that

$$f(2)=6,\ f'(2)=4,\ f''(2)=-7,\ f'''(2)=8$$

(a) Write the third-degree Taylor polynomial for f about $x = 2$, and use it to approximate $f(2.3)$.

$$P_{(3)}(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2!} + \frac{f'''(2)(x-2)^3}{3!}$$

$$P_{(3)}(x) = 6 + 4(x-2) - \frac{7(x-2)^2}{2!} + \frac{8(x-2)^3}{3!}$$

$$P_{(3)}(2.3) = 6 + 4(2.3-2) - \frac{7(2.3-2)^2}{2!} + \frac{8(2.3-2)^3}{3!}$$

$$P_{(3)}(2.3) = 6.921$$

$$f(2.3) \approx 6.921$$

$$f(2.3) \approx P_{(3)}(x)$$

Ex. 4

The function f has derivatives of all orders for all reall numbers x. Assume that

$f(2)=6, f'(2)=4, f''(2)=-7, f'''(2)=8$

$|f^{(4)}(x)| \le 9$

(b) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \le 9$ for all x in the closed interval [2, 2.3]. Use the Lagrange error bound on the the apporximation of f (2.3) found in part (a) to find an interval [a, b] such that

x: 2.3
c: 2
n: 3

$$E_{max} \le \left| \frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!} \right|$$

$$\left| \frac{\cancel{f^4(z)}(2.3-2)^4}{4!} \right|$$

$$\frac{9(.3)^4}{4!} = .0030375$$

$6.921 \pm .0030375$

$6.918 \le f(2.3) \le 6.924$

$[6.918, 6.924]$

(c) Could $f(2.3)$ equal 6.922? Show why or why not.

Yes, it's in the interval

Ex. 5

Suppose a function f is approximated with a fourth-degree Taylor polynomial about x = 1. If the maximum value of the fifth derivative between x = 1 and x = 3 is .01, that is, $|f^{(5)}(x)| \le .01$ then the maximum error incurred using this approximation to compute f (3) is

(A) 0.054 (B) 0.0054 (C) 0.26667 (D) 0.02667 (E) 0.00267

x = 3
c = 1
n = 4

$$\frac{f^5(z)(3-1)^5}{5!}$$

$$\frac{.01(2)^5}{5!}$$

Ex. 6

Selected values of f and its first four derivatives are indicated in the table below. The function f and its derivatives are decreasing on the interval 0<x<4.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
3	12	-31	-48	-81	-16

A.) write the third-degree Taylor polynomial for f about x=3 and use it to approximate f(3.1).

$$P_{(3)}(x) = f(3) + f'(3)(x-3) + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(3)(x-3)^3}{3!}$$

$$P_{(3)}(x) = 12 - 31(x-3) - \frac{48(x-3)^2}{2!} - \frac{81(x-3)^3}{3!}$$

$$P_{(3)}(3.1) = 8.6465$$

$$f(3.1) \approx P_{(3)}(3.1)$$

$$f(3.1) \approx 8.6465$$

B.) use the LAGRANGE error bound to show that the third-degree Taylor polynomial for f about x=3 approximates f(3.1) with error less than $7x10^{-5}$

.00007

$x=3.1$
$c=3$
$n=3$

$$E_{max} \le \left| \frac{f^4(z)(3.1-3)^4}{4!} \right|$$

$$\left| \frac{-16(.1)^4}{4!} \right|$$

6.6667 E-5 $\times 10^{-5}$

.0000666666

$.0000666 < 7 \times 10^{-5}$

Ex. 7

non calculator

The third-degree Taylor polynomial for h about x = -2 is given by:

$$P_3(x) = 4 + 9(x+2) - \frac{1}{3}(x+2)^2 + \frac{5}{6}(x+2)^3$$

$$P_{(3)}(x) = h(-2) + h'(-2)(x+2) + \frac{h''(-2)(x+2)^2}{2!} + \frac{h'''(-2)(x+2)^3}{3!}$$

A) find $h'''(-2)$

5

$$\frac{5}{6} = \frac{h'''(-2)}{3!}$$

B) Does h have a relative maximum, relative minimum, or neither at x = -2?

$f'(x) = 0$

$f'(x) \neq 0$

neither

C) The fourth derivative of h satisfies the inequality $|f^{(4)}(x)| \leq 10$ for all x in the closed interval [-2, 0]. Use the Lagrange error bound on the approximation to f(0) to show $|P_3(0) - f(0)| < 7$

$x = 0$
$c = -2$
$n = 3$

$$E_{max} \leq \left| \frac{f^{4}(z)(0+2)^4}{4!} \right|$$

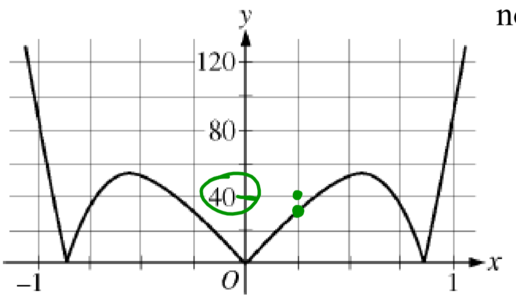
$$\frac{10(2)^4}{4!}$$

$$\frac{10 \cdot 2 \cdot \cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1} = \frac{20}{3}$$

$$\frac{20}{3} < 7$$

Ex. 8 2011 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

non calculator



Graph of $y = |f^{(5)}(x)|$

(d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

$x : 1/4$
$c : 0$
$n : 4$

$$\left| \frac{\cancel{f^{5}(z)}\left(\frac{1}{4} - 0\right)^5}{5!} \right|$$

$$\frac{40\left(\frac{1}{4}\right)^5}{5!} = \frac{\cancel{40} \quad 1}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \quad \cancel{5} \cdot \cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1}$$

64
16

Error/ Remainder

error = actual - approximation

2 scenarios

- If series **alternates** and **converges**... then $|\text{error}| <$ next term evaluated at x = c

- Lagrange Error Bound If series does not alternate, but still converges then $|\text{error}| <$ next term

$$E_{\max} \leq \left| \frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!} \right|$$

con't on next slide

Lagrange error continued...

$$E_{\max} \leq \left| \frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!} \right|$$

$f^{n+1}(z)$ is a maximum value that $f^{n+1}(z)$ takes on an interval containing some value between x_o and c