Does this converge or diverge

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)^n\cancel{(n+1)}}{n^n}\cdot\frac{\cancel{n!}}{\cancel{(n+1)}\cancel{n!}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)^n}{n^n}\right|$$

$$\lim_{n\to\infty}\left|\left(\frac{n+1}{n}\right)^n\right|$$

$$\lim_{n\to\infty}\left|\left(1+\frac{1}{n}\right)^n\right|$$

e

$e > 1$ diverges by Ratio Test

9.8 Power Series

Read pages 659 - 660

- What is a power series?
- How do you know where the series is centered?
- What are the 3 types of domains for a power series?
- What is the difference between the radius of convergence and the domain for convergence? Draw a picture.

domain ~ point
R = 0

domain ~ interval
R = interval/2

domain ~ all real numbers
R = ∞

Summary
Power Series Intervals of Convergence

1) Use Ratio Test to find one of three possibilities...

   a) converge only at a point, otherwise diverge

   b) always converges

   c) if inconclusive set expression $-1 < x < 1$

   Check endpoints **separately**

	Domain of Convergence	**Radius** of Convergence
ratio test lim<1 doesn't matter what x is	all real numbers $(-\infty,\infty)$	$R = \infty$
ratio test lim>1 doesn't matter what x is	only one point (its center) $\{2\}$ $\{0\}$ $\{-\pi\}$ etc.	$R = 0$
ratio test lim=1	**interval** such as $(-1,7]$ $[-1,7)$ $[-1,7]$ $(-1,7)$	(right-left)/2 $(7+1)/2 = 4$

Power Series
Radius of Convergence

- Find the interval of convergence
- Compute its length
- Divide by 2

next slide for examples

Ex. 1

$$\sum_{n=0}^{\infty} \frac{(2n)!x^{2n}}{n!}$$

$$\lim_{n\to\infty} \left| \frac{(2(n+1))!\,x^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{(2n)!\,x^{2n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(2n+2)!\,x^{2n+2}}{(n+1)!} \cdot \frac{n!}{(2n)!\cdot x^{2n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{(2n)!}} \cdot \frac{\cancel{n!}}{(n+1)\cancel{(n)!}} \cdot \frac{\cancel{x^{2n}}\cdot x^2}{\cancel{x^{2n}}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)\,x^2}{n+1} \right|$$

$\infty > 1$ Only way to make converge

Domain $x=0$ Radius 0

ex. $\sum_{n=1}^{\infty} \frac{(2n)!}{n!}(x-4)^{2n}$

Domain $x=4$ Radius 0