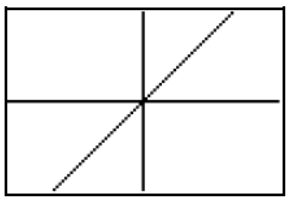
9.7 Taylor Polynomials and Approximations

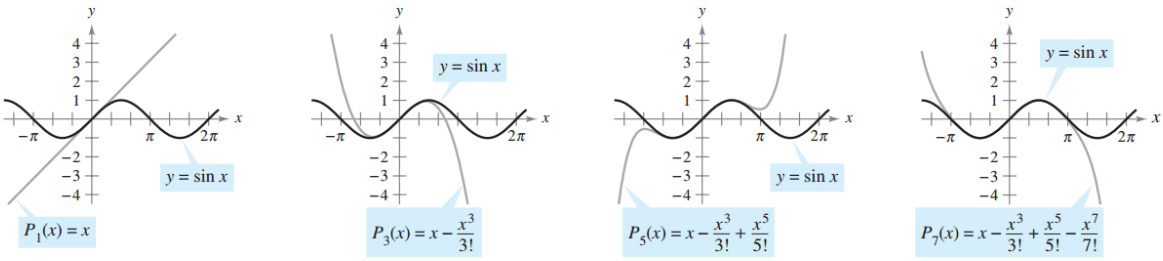
do not take notes!!!
just observe...we will fill in notes later

Power Series and Taylor Polynomials
An Introduction with problems

I. Consider the function shown below.
What function do you think it is? _____



(Larry Peterson Packet)



As n increases, the graph of P_n more closely resembles the sine function.

Taylor Polynomials

$$P_{(n)}(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \ldots \frac{f^n(c)(x-c)^n}{n!}$$

MacLaurin Polynomials
is the same thing except
c = 0

$$P_{(n)}(x) = f(0) + f'(0)(x) + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots \frac{f^n(0)x^n}{n!}$$

Three Parts-do you see it?

$$\frac{f^n(c)(x-c)^n}{n!}$$

derivative

$$\frac{f^n(c)(x-c)^n}{n!}$$

coefficient

variable

1

Ex. 1 $c=0$

Find a 4th degree MacLaurin Polynomial for

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f^3(x) = \sin x$

$f^4(x) = \cos x$

$f(0) = \cos 0 = 1$

$f'(0) = -\sin 0 = 0$

$f''(0) = -\cos 0 = -1$

$f^3(0) = \sin 0 = 0$

$f^4(0) = \cos 0 = 1$

$$P_{(n)}(x) = f(0) + f'(0)(x) + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots \frac{f^n(0)x^n}{n!}$$

$$P_4(x) = 1 + 0x - \frac{1x^2}{2!} + \frac{0x^3}{3!} + \frac{1x^4}{4!}$$

$$P_{(4)}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\cos x \approx P_{(4)}(x)$$

Ex. 2

c = 0

Find a 3rd degree MacLaurin Polynomial and nth term for

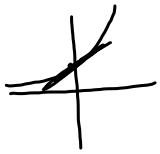
$f(x) = e^x$

$f(0) = e^0 = 1$

$f'(x) = e^x$ $f'(0) = 1$

$f''(x) = e^x$ $f''(0) = 1$

$f'''(x) = e^x$ $f'''(0) = 1$



$$P_{(n)}(x) = f(0) + f'(0)(x) + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots \frac{f^n(0)x^n}{n!}$$

$P_{(3)}(x) = 1 + 1x + \frac{1x^2}{2!} + \frac{1x^3}{3!}$

$f(x) \approx P_{(3)}(x)$

$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$e^x \approx P_{(3)}(x)$

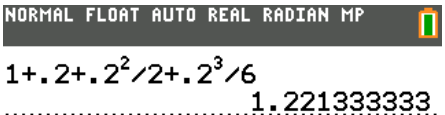
Let's use $P_3(x)$ to approximate f(.2)

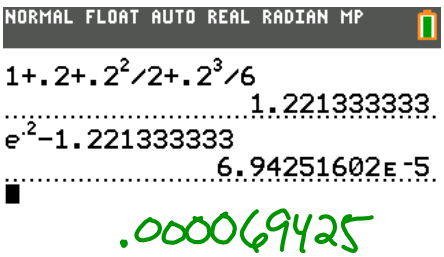
$f(x) = e^x$

$P_{(3)}(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$f(.2)$

$e^{.2} \approx 1 + .2 + \frac{(.2)^2}{2!} + \frac{(.2)^3}{3!}$





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