9.6 Ratio and Root Tests

Review Factorials

$$\frac{\cancel{n!}}{(n+1)\cancel{n!}} \qquad \frac{1}{n+1}$$

$$\frac{n!}{(n+1)!} \sim to \ \frac{7!}{8!}$$

$$\frac{\cancel{7!}}{8 \cdot \cancel{7!}} \qquad \frac{1}{8}$$

$\boxed{\text{Ratio Test}}$

big for 9.8

$$\sum_{n=1}^{\infty} a_n \qquad \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

<1	conv absolutely
>1 or ∞	diverges
$=1$	inconclusive

useful for factorials and exponents

Ex. 1

$$\sum_{n=1}^{\infty} \frac{n^3}{2^n}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^3}{2^{n+1}}}{\frac{n^3}{2^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{2^{n+1}} \cdot \frac{2^n}{n^3} \right|$$

$$\lim_{n\to\infty} \frac{\cancel{(n+1)^3} \cdot 2^n}{\cancel{n^3} \cdot 2^{n+1}}$$

$$\lim_{n\to\infty} 1 \frac{\cdot \cancel{2^n}}{\cancel{2^n} \cdot 2^1}$$

$$\frac{1}{2} < 1$$

Conv. absolutely by Ratio Test

Ex. 2

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n)^5}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(2(n+1))!}{(n+1)^5}}{\frac{(2n)!}{n^5}} \right|$$

$$\lim_{n\to\infty} \frac{(2n+2)!}{(n+1)^5} \cdot \frac{n^5}{(2n)!}$$

$$\lim_{n\to\infty} \frac{\cancel{n^5}}{\cancel{(n+1)^5}} \frac{(2n+2)!}{(2n)!}$$

$$\lim_{n\to\infty} 1 \cdot \frac{(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{(2n)!}}$$

$$\infty$$

diverges by Ratio Test

Ex. 3

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n!}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} 3^{n+1}}{(n+1)!}}{\frac{(-1)^n 3^n}{n!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} 3^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^n 3^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{(-1)^n} \cdot \frac{3^{n+1}}{3^n} \cdot \frac{n!}{(n+1)!} \right|$$

$$\lim_{n\to\infty} \frac{\cancel{3^n} \cdot 3^1}{\cancel{3^n}} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}}$$

$$\lim_{n\to\infty} \frac{3}{n+1}$$

$0 < 1$ Conv. absolutely by Ratio Test

Ex. 4

show 1/n and 1/n^2

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

$$\lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)^p}}{\frac{1}{n^p}} \right|$$

$$\lim_{n\to\infty} \frac{1}{(n+1)^p} \cdot \frac{n^p}{1}$$

$$\lim_{n\to\infty} \frac{n^p}{(n+1)^p}$$

1 inconclusive

not that much

Root Test

$$\sum_{n=1}^{\infty} a_n \qquad \lim_{n\to\infty} \sqrt[n]{|a_n|}$$

<1	conv absolutely
>1 or ∞	diverges
$=1$	inconclusive

****
useful for n^{th} powers
****

Ex. 5

$$\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^n$$

$$\lim_{n\to\infty} \left|\sqrt[n]{\left(\frac{2n}{n+1}\right)^n}\right|$$

$$\lim_{n\to\infty} \frac{2n}{n+1}$$

$2 > 1$ diverges by Root test

Ex. 6

$$\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{\ln n}{n}\right)^n}$$

$$\lim_{n\to\infty}\frac{\ln n}{n}$$

$$\lim_{n\to\infty}\frac{\frac{1}{n}}{1}$$

$$\lim_{n\to\infty}\frac{1}{n}$$

$0<1$ conv. absolutely by Root test

Ex. 7

$$\sum_{n=1}^{\infty}\frac{(n!)^n}{(n^n)^2}=\sum_{n=1}^{\infty}\frac{(n!)^n}{(n^2)^n}=\sum_{n=1}^{\infty}\left(\frac{n!}{n^2}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n!}{n^2}\right)^n}$$

$$\lim_{n\to\infty}\frac{n!}{n^2}$$

very top heavy

∞ diverge by Root test

Ex. 8 ☆☆☆ see a lot more in 9.8

Find the values of x for which the series converges.

$$\sum_{n=1}^{\infty}\left(\frac{x+1}{4}\right)^n$$

$$\left|\frac{x+1}{4}\right| < 1$$

$$-1 < \frac{x+1}{4} < 1$$

$$-4 < x+1 < 4$$

$$\quad -1 \qquad -1 \quad -1$$

$$-5 < x < 3$$