Sum of an Alternating Series

If $\sum_{n=1}^{\infty} (-1)^n b_n$

is a convergent alternating series, then

$$|R_n| = |S - S_n| \le b_{n+1}$$

Remainder (Error) — Series — Approximation

Error is **at most** size of first neglected term.

Ex.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

lim ⟶ 0
decreasing } converges

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots$$

$$1 - \frac{1}{2} + \frac{1}{3}$$

Correct approximation within an error of at most 1/4 (.25)

Now if add S_4 :

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$$

Now correct within 1/5 (.2) because first neglected term is 1/5

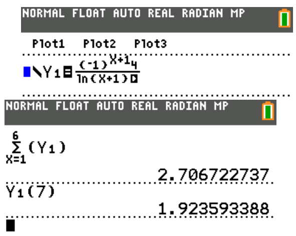
Error is *at most* the size of the first neglected term.

for Alternating Series

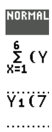
Ex.

Approximate the sum of the series using the first 6 terms.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}4}{\ln(n+1)}$$



Sum ≈ 2.708 ± 1.924



Ex.
Determine the number of terms required to approximate the sum of the series with an error of less than .001. Then estimate the sum.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n!)}$$

.0010000000

less than .001: means it's the first value that has zero in thousandths place i.e. .000x

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = \frac{(-1)^X}{(2X!)}$

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = \frac{(-1)^X}{(2X!)}$ ▸Dec

Y_2=
Y_3=
Y_4=
Y_5=
Y_6=
Y_7=

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-1	ERROR
0	0.5
1	-0.5
2	0.25
3	-0.083
4	0.0208
5	-0.004
6	6.9E-4
7	-1E-4
8	1.2E-5
9	-1E-6

X= -1

1st omitted (neglected)

6.9E-4

6.9×10^{-4}

.00069

5 terms

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
4	1/48
5	-1/240
6	1/1440
7	-1E-4
8	1.2E-5
9	-1E-6
10	1.4E-7

X=8

Ex.
One more approximating the sum correct to 4 decimal places (Need 4 zeros). .0000_

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{8^n}$$

$\frac{1}{8^n} = \left(\frac{1}{8}\right)^n$

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = \frac{(-1)^X}{(8^X)}$

Y_2=
Y_3=
Y_4=
Y_5=
Y_6=
Y_7=

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
1	-0.125
2	0.0156
3	-0.002
4	2.4E-4
5	-3E-5
6	3.8E-6
7	-5E-7
8	6E-8
9	-7E-9
10	9E-10
11	-1E-10

X=1

$-3E-5$

-3×10^{-5}

-.00003

NORMAL FLOAT AUTO REAL RADIAN MP

$\sum_{X=1}^{4}(Y_1)$

-0.1110839844

$-.111 \pm .00003$

Sum of an Alternating Series

If $$\sum_{n=1}^{\infty}(-1)^n b_n$$

is a convergent alternating series, then

$$|R_n| = |S - S_n| \le b_n + 1$$

Remainder (Error) — R_n

Series — S

Approximation — S_n

Error is **at most** size of first neglected term.