9.5 Alternating Series

easiest of all to deal with

$n=1$ $n=2$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\ldots$$

refers to the signs

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$$

If trying to confuse you...
start writing out some terms

Be careful!!

$$\sum_{n=1}^{\infty}(-1)^{2n}\frac{1}{n}$$

In general, look for -1 raised to a power

$$\sum_{n=1}^{\infty}(-1)^{n}b_n$$

Just have to show 2 things for series to <u>converge</u>:

Alternating Series will Converge if...

1. $$\lim_{n\to\infty} b_n = 0$$

2. $$b_{n+1} \le b_n$$

$\boxed{\text{(}b_n\text{'s are decreasing) just has to eventually decrease, not all}}$

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if both don't happen, then series diverges

Ex. 1

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

1. Alternating? ✓

2. decreasing? ✓

3. $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

Conv. by AST

conditional

if $(-1)^n$ wasn't there, then harmonic and diverges (p - series)

Ex. 2

$(-1)^{n+1} \cdot \frac{1}{n^2+1}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$$

Alt ✓

$\frac{1}{n^2+1}$ dec ✓

lim→0 ✓

Conv. by AST

absolutely conv. b/c $\sum \frac{1}{n^2+1}$ LCT to $\frac{1}{n^2}$

Ex. 3

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n^2+5}$$

Alt ✓

$\frac{n^2}{n^2+5}$ dec

~~lim→0~~

div. by AST

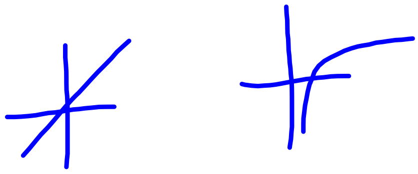
Ex. 4

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{\ln n}{n}\right)$$

Alt. ✓

$\lim_{x\to\infty} \Rightarrow 0$ $\qquad \frac{\frac{1}{x}}{1} = \frac{1}{x}$

dec.



Conv. by AST

Ex. 5

$$\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^{3/4}}$$

n=1	n=2	n=3
-1	1	-1

Alt.

$\frac{1}{n^{3/4}}$ dec ✓

$\lim_{n\to\infty} \to 0$ ✓

AST conv.

Conditional by $\sum \frac{1}{n^{3/4}}$ div. by p-series

white boards...to finish day 1

Alternating Series Test

Converge

Diverge

Absolute Convergence

Conditional Convergence

Absolute Convergence	Conditional Convergence
If $\sum_{n=1}^{\infty} \lvert a_n \rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.	If $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} \lvert a_n \rvert$ diverges

Ex.

Determine if the series converges conditionally or absolutely, or if it diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

Alt. ✓

$\frac{1}{n+1}$ dec ✓ lim→0 ✓

Conv. by AST

conditional

$\sum \frac{1}{n+1}$ div. by LCT to $\sum \frac{1}{n}$

Ex.

Determine if the series converges conditionally or absolutely, or if it diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$$

Alt ✓

$\frac{1}{n^{3/2}}$ dec lim→0

Conv. by AST

absolutely

$\sum \frac{1}{n^{3/2}}$ conv. by pseries

Ex.

Determine if the series converges conditionally or absolutely, or if it diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n(2n+3)}{n+10}$$

Alt ✓

$\frac{2n+3}{n+10}$ — $\lim \neq 0$

div. by AST

Sum of an Alternating Series

If $\sum_{n=1}^{\infty} (-1)^n b_n$

is a convergent alternating series, then

$$|R_n| = |S - S_n| \leq b_{n+1}$$

Remainder (Error)

Series

Approximation (estimate)

Error is **at most** size of first neglected term.

Ex.

$$\lim_{n\to\infty}(-1)^{n+1}\frac{1}{n}$$

lim ⟶ 0
decreasing } converges

$$\underbrace{1-\frac{1}{2}+\frac{1}{3}}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\dots$$

$$1-\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$$

Correct approximation within an error of at most 1/4 (.25)

Now if add S_4 :

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$$

error

Now correct within 1/5 (.2) because first neglected term is 1/5

Error is **<u>at most</u>** the size of the first neglected term.

Remainder

(Not using)

for Alternating Series