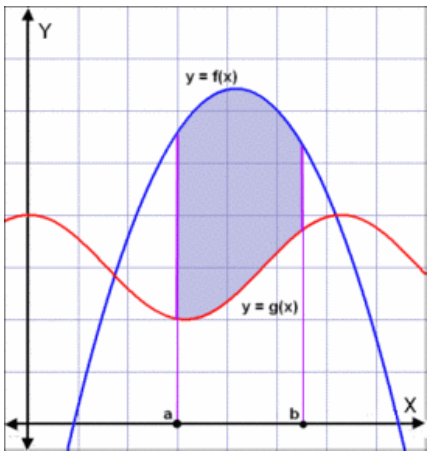
7 Applications of Integration

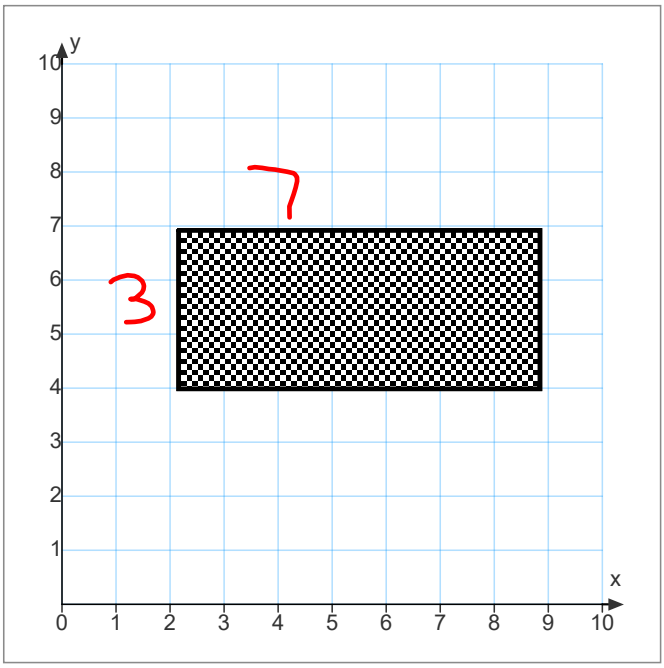
7.1 Area of a Region Between Two Curves



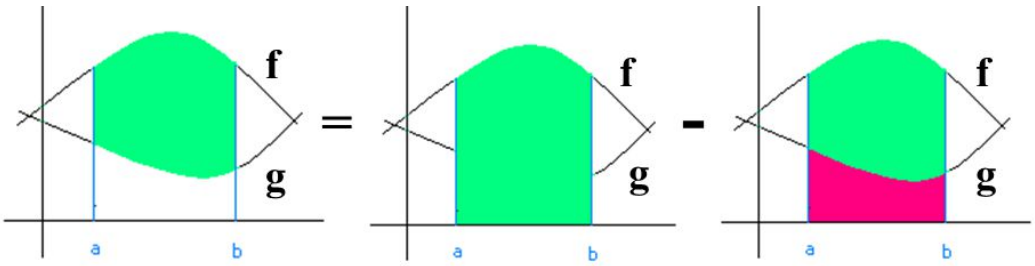
What's the area of the rectangle?

21 units²

7-4
3



9-2
7



Area of region between f and g = **Area of region under f(x)** - **Area of region under g(x)**

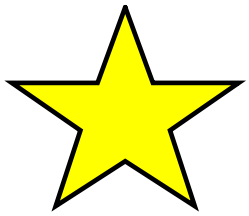
$$\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

=Top function - Bottom function

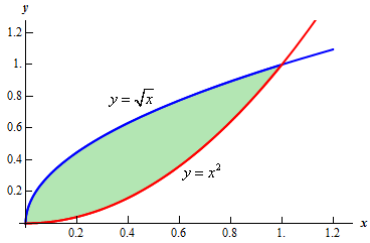
Area of a Region Between Two Curves

If f and g are continuous on[a, b] and g(x) ≤ f(x) for all x in [a, b], then the area of the region bounded by the graphs of f and g and the vertical lines x = a and x = b is

$$A=\int_a^b \left[f(x)-g(x)\right]dx$$



Finding the Area of a Region Between Two Curves



$f(x)=\sqrt{x}$

$g(x)=x^2$

$\sqrt{x}=x^2$

$x=x^4$

$0=x^4-x$

$0=x(x^3-1)$

$\int_0^1 \sqrt{x}-x^2\,dx$

$\int_0^1 x^{1/2}-x^2\,dx$

$\frac{2}{3}x^{3/2}-\frac{x^3}{3}\Big]_0^1$

$F(1)-F(0)$

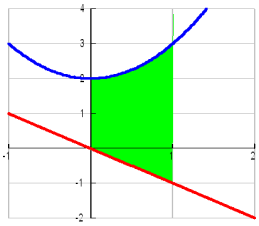
$\left(\frac{2}{3}(1)^{3/2}-\frac{1}{3}\right)-(0-0)$

$\frac{2}{3}-\frac{1}{3}$

$\frac{1}{3}$

Finding the Area of a Region Between Two Curves

$f(x)=x^2+2$



$g(x)=-x$

$$\int_0^1 x^2+2-(-x)\,dx$$

$$\int_0^1 x^2+x+2\,dx$$

$$\frac{x^3}{3}+\frac{x^2}{2}+2x\Big]_0^1$$

$F(1) - F(0)$

$$\left(\frac{1}{3}+\frac{1}{2}+2\right)-0$$

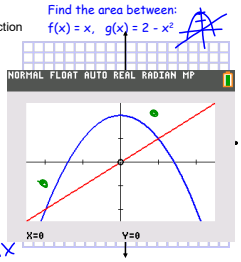
$$\frac{2}{6}+\frac{3}{6}+\frac{12}{6}$$

$$\frac{17}{6}$$

Area of a Region Between Intersecting Curves

1. Draw a graph
2. Find the points of intersection
3. Integrate
4. Check on calculator!

Find the area between:
$f(x) = x$, $g(x) = 2 - x^2$



2. $x = 2-x^2$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2,1$

$$\int_{-2}^{1} -x^2+2-x\,dx$$

$$-\frac{x^3}{3}+2x-\frac{x^2}{2}\Big]_{-2}^{1}$$

$F(1)$

$$\left(-\frac{(1)^3}{3}+2(1)-\frac{(1)^2}{2}\right)-\left(-\frac{(-2)^3}{3}+2(-2)-\frac{(-2)^2}{2}\right)$$

$$-\frac{1}{3}+2-\frac{1}{2}-\left(\frac{8}{3}-4-2\right)$$

$$-\frac{1}{3}+2-\frac{1}{2}-\frac{8}{3}+6$$

$$-\frac{9}{3}+8-\frac{1}{2}$$

$$-3+8-\frac{1}{2}$$

$$5-\frac{1}{2}$$

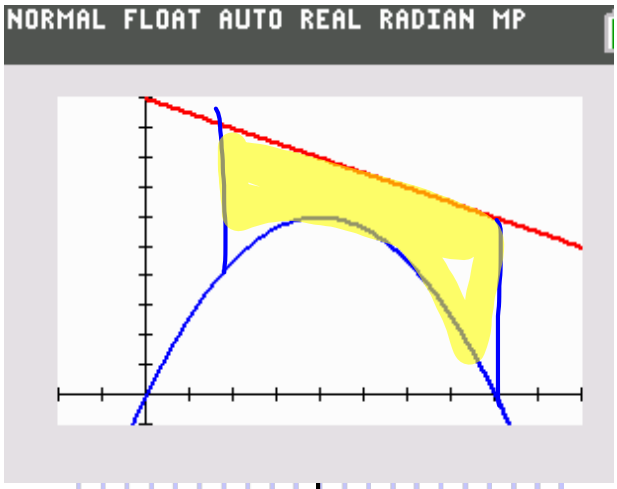
$$4\frac{1}{2}=\frac{9}{2}$$

Find the area between

$$y = -\frac{3}{8}x(x-8)$$

$$y = 10 - \frac{1}{2}x$$

on [2, 8]



end day 1