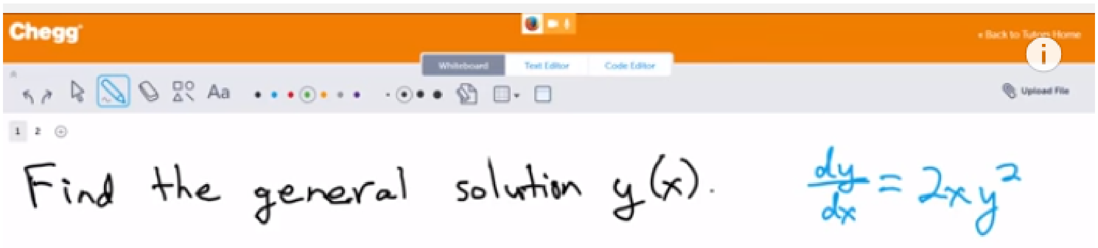
# 6.2 Growth and Decay




https://www.youtube.com/watch?v=mKe8qAFrWPs (3:32)

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$(\cos y + 2)\dfrac{dy}{dx} = 2x$ and $y(1) = 0.$

**What is $x$ when $y = \pi$?**

https://www.khanacademy.org/math/ap-calculus-ab/ab-differential-equations-new/ab-7-7/v/using-particular-solution-to-separable-differential-equation

(3:17)

## Exponential models & differential equations (Part 1)

https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/exponential-models-diff-eq/v/modeling-population-with-simple-differential-equation (7:41)

## Exponential models & differential equations (Part 2)

https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/exponential-models-diff-eq/v/particular-solution-given-initial-conditions-for-population (4:52)

## Worked example: exponential solution to differential equation

https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/exponential-models-diff-eq/v/exponential-solution-to-differential-equation (4:26)

Ex. 1

a) Solve the differential equation:

$$dx \cdot \frac{dy}{dx} = (4 - x)\,dx$$

$$\int 1\,dy = \int (4-x)\,dx$$

$$y = 4x - \frac{x^2}{2} + C$$

b) Solve the differential equation:

$y' = x(1 + y)$

$dx \cdot \frac{dy}{dx} = x(1+y)\,dx$ Separation of variables

$\frac{dy}{(1+y)} = \frac{x\cancel{(1+y)}\,dx}{\cancel{(1+y)}}$

$\int \frac{1}{1+y}\,dy = \int x\,dx$

$u = 1+y$

$du = dy$

$\int \frac{1}{u}\,du$

$\ln|1+y| = \frac{x^2}{2} + c$

$e^{\ln|1+y|} = e^{\frac{x^2}{2}+c}$

$1+y = e^{x^2/2} \cdot e^{c}$

$1+y = C e^{x^2/2}$

$y = Ce^{x^2/2} - 1$

c) Solve the differential equation:

$$\frac{dy}{dt} = \frac{1}{3}t$$

- find the general solution
- graph particular solution that has (0, 10) as a point.

$\int dy = \int \frac{1}{3} t\,dt$

$y = \frac{1}{3} \cdot \frac{t^2}{2} + c$

$y = \frac{t^2}{6} + c$ → general solution

$10 = \frac{0^2}{6} + c$

$10 = c$

$y = \frac{t^2}{6} + 10$

d) The rate of change of P with respect to t is proportional to 10 - t.
Find the function.

$$\frac{dP}{dt} = K(10-t)$$

$$\int dP = \int K(10-t)\,dt$$

$$P = K\int 10-t\,dt$$

$$P = K\left(10t - \frac{t^2}{2} + C\right)$$

$$= K\left(10t - \frac{t^2}{2}\right) + C$$

e) The rate of change of N (w/ Respect to time) is proportional to N.
When t = 0, N = 250 and when t = 1, N = 400.
What is the value of N when t = 4?

$$\frac{dN}{dt} = KN$$

$$\frac{dN}{N} = \frac{KN\,dt}{N}$$

Separation of variables

$$\int \frac{1}{N}\,dN = \int K\,dt$$

$$\ln|N| = Kt + C$$

$$e^{\ln|N|} = e^{Kt+C}$$

$$N = e^{Kt}\cdot e^{C}$$

$t=0$ $N=250$
$t=1$ $N=400$
$t=4$ $N=?$

$$N = Ce^{Kt}$$

$$250 = Ce^{K(0)}$$

$$250 = Ce^{0}$$

$$250 = C$$

$$N = 250e^{Kt}$$

$$400 = 250e^{K(1)}$$

$$\frac{400}{250} = e^{K}$$

$$\frac{8}{5} = e^{K}$$

$$\ln 8/5 = K$$

$$N = 250e^{(\ln 8/5)t}$$

$t = 4$

Theorem 6.1

If $y' = ky$, then $y = Ce^{kt}$

Let's prove it!

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = \frac{ky\,dt}{y}$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln|y| = kt + C$$

$$e^{\ln|y|} = e^{kt+C}$$

$$y = e^{kt} \cdot e^{C}$$

$$y = Ce^{kt}$$

**Stand and Deliver**

## Growth and Decay

6.2

$$y' = ky$$

The rate of change of y is proportional to y.

$$y = Ce^{kt}$$

C = initial amount

$k$ = proportionality constant

Word Problems in Groups!
In this order: 64, 71, 71

64. ***Bacteria Growth*** The number of bacteria in a culture is increasing according to the law of exponential growth. There are 125 bacteria in the culture after 2 hours and 350 bacteria after 4 hours.

(a) Find the initial population.

(b) Write an exponential growth model for the bacteria population. Let $t$ represent time in hours.

(c) Use the model to determine the number of bacteria after 8 hours.

(d) After how many hours will the bacteria count be 25,000?

$y = Ce^{kt}$

$125 = Ce^{2k}$ $\qquad$ $350 = Ce^{4k}$

$C = \frac{125}{e^{2k}}$

$350 = \left(\frac{125}{e^{2k}}\right)e^{4k}$

$\frac{350}{125} = \frac{125e^{2k}}{125}$

$\frac{350}{125} = e^{2k}$

$\frac{\ln\left(\frac{350}{125}\right)}{2} = k$

do

The rate of change of a (with respect to b) is inversely proportional to the square root of b. Find the function.

$\frac{dA}{db} = \frac{1}{k}\sqrt{b}$

$\frac{dA}{db} = \frac{1}{k} b^{+1/2}$

$\int dA = \int \frac{1}{k} b^{1/2}\, db$

$A = \frac{1}{k}\frac{2}{3} b^{3/2} + C$

$A = \frac{2}{3k} b^{3/2} + c$