5.3 Inverse Functions f^{-1}

Objective: You will be able to:

- verify inverse functions
- determine if a function has an inverse
- find the derivative of an inverse function

Warm Up

Inverse Functions-

To **find** an inverse function:

To **verify** inverses:

Graphically...

Vertical Line Test:

Horizontal Line Test:

Inverse Functions- switch x and y

domain and range

To **find** an inverse function:

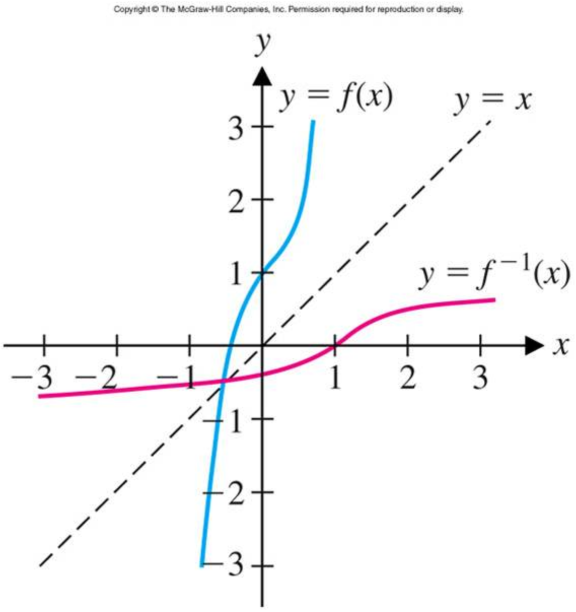
- switch x and y and solve for y

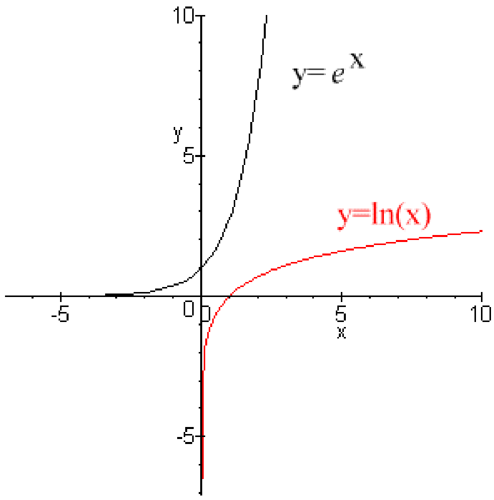
To **verify** inverses:

- $f(g(x)) = x$
- $g(f(x)) = x$

Inverse Functions- **Graphically**

inverse graphs are reflections about the line y = x





Vertical Line Test

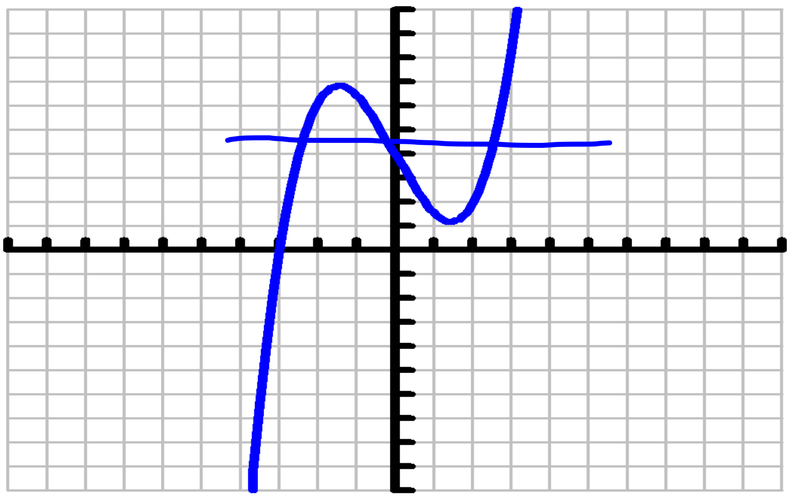
y is a **function** of x iff (if and only if) no vertical line intersects the graph at more than one point.

Horizontal Line Test

to see if it has an inverse
"one to one"
must pass the test to have an inverse

Strictly monotonic

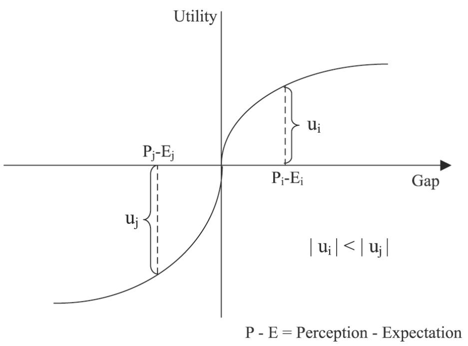
1. Is this a function? Yes
2. Does it have an inverse? NO

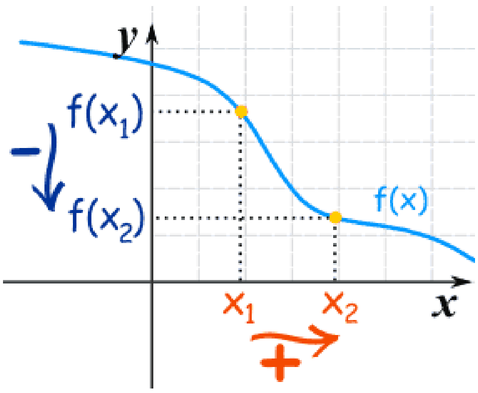


Strictly Monotonic Function:

increasing or decreasing
over the entire interval

and ∴ "one to one"





Theorem: The Existence of an Inverse Function

1. if and only if it is one-to-one

2. if strictly monotonic over its entire domain,
   then it is one-to-one and therefore has
   an inverse

precalc way f(g(x))=g(f(x))
calc way compare deriv.

Ex. 1

Show that f and g are inverses

$$f(x) = x^3 \qquad g(x) = \sqrt[3]{x}$$

Ex. 2

Decide if each is "one to one"

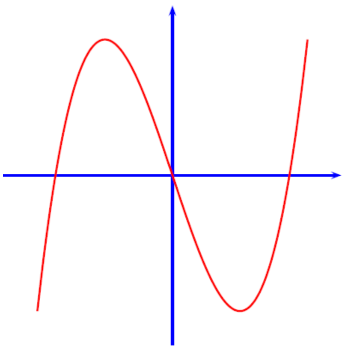
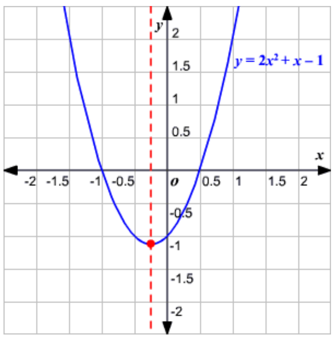
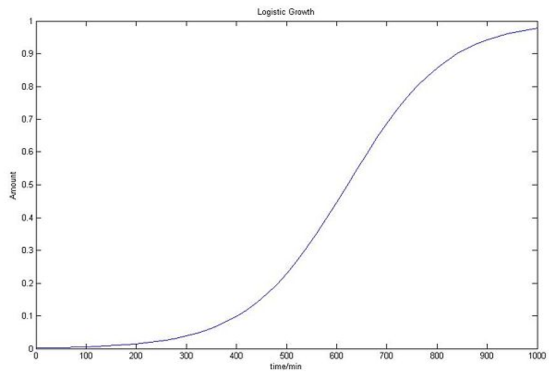


Figure 3: Cubic function.





Ex. 3

Find the inverse of

1. $y = 3x + 1$

2. $y = x^2$

Ex. 4

Find the inverse of $f(x) = x^2 + 3$

why do we need...

$x \geq 3$

Graphs of inverses have

RECIPROCAL SLOPES (DERIVATIVES)

(note: not opposite reciprocals)

Stand and Deliver

Derivative of an Inverse Function

5.3

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

if given $f^{-1}(b)$

set f(x)= b, solve for **x**

evaluate that **x** in f '(**x**)

then do reciprocal to find deriv. of inverse function f

Geometers Sketchpad:

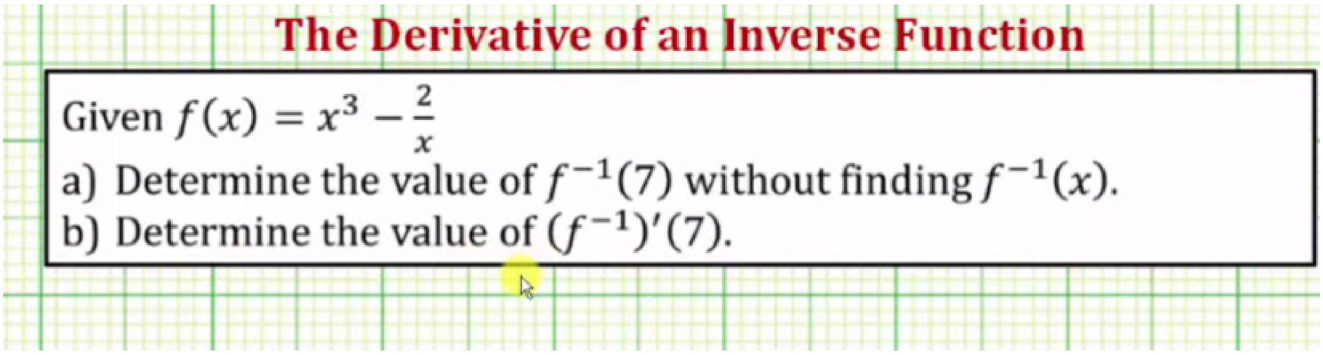
Inv Function Der.gsp

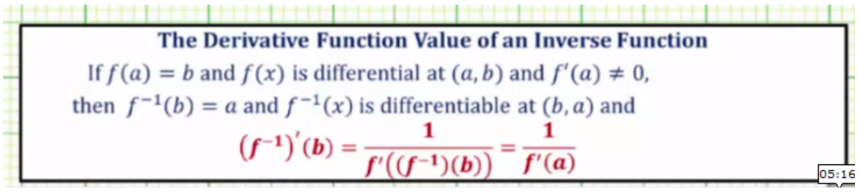
Theorem: Derivative of an inverse function

If f has an inverse function *g*, then *g* is differentiable at any *x* for which $f'(g(x)) \neq 0$, and

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0$$

https://www.youtube.com/watch?v=mhn1kQFGR44





https://www.youtube.com/watch?v=UOiTZaKN_-I&feature=youtu.be

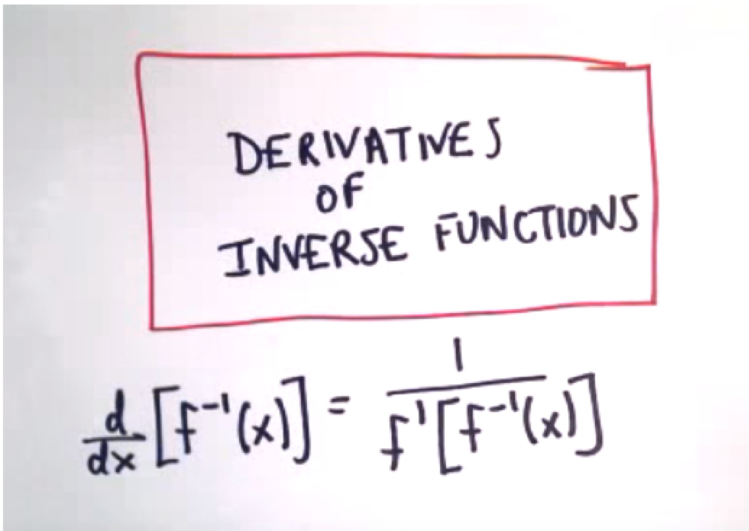
The Derivative of an Inverse Function

Given $f(x) = \sin(x)$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

a) Determine the value of $f^{-1}\left(\frac{\sqrt{3}}{2}\right)$ without finding $f^{-1}(x)$.

b) Determine the value of $(f^{-1})'\left(\frac{\sqrt{3}}{2}\right)$.

https://www.youtube.com/watch?v=RKfGMX0pn2k



29. The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

f^{-1}

$f(0)=2$
$(0,2)$

$f^{-1}(2)=0$
$(2,0)$

(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 4

$f'(x)=3x^2+4$

$f'(0)=3(0)^2+4=4$

$(f^{-1})'(2)=\frac{1}{4}$

Alternatively, one can use the following formula that follows from the same chain rule calculation as above when starting with $f\left(f^{-1}(x)\right) = x$:

$$\left.\frac{d}{dx} f^{-1}(x)\right|_{x=2} = \frac{1}{f'\left(f^{-1}(2)\right)} = \frac{1}{f'(0)} = \frac{1}{3 \cdot 0^2 + 4} = \frac{1}{4}.$$

Inv Function Der.gsp