

Ex. 4

$$\int \frac{x+1}{x^2+2x} dx$$

$$\frac{1}{2} \int \frac{1}{u} du$$

$$\frac{1}{2} \ln|u| + c$$

$$\frac{1}{2} \ln|x^2+2x| + c$$

$$\ln \sqrt{x^2+2x} + c$$

$$u = x^2 + 2x$$
$$du = (2x + 2) dx$$
$$du = 2(x+1) dx$$
$$\frac{1}{2} du = (x+1) dx$$

Ex. 5

Long Division  
(Top Heavy or Tied)

$$\int \frac{2x^2 + 7x - 3}{x - 2} dx$$

$$\begin{array}{r}
 2x + 11 + \frac{19}{x-2} \\
 x-2 \overline{) 2x^2 + 7x - 3} \\
 \underline{-(2x^2 - 4x)} \phantom{-3} \\
 11x - 3 \\
 \underline{-11x + 22} \\
 19
 \end{array}$$

$$\int \frac{2x^2 + 7x - 3}{x - 2} dx = \int 2x + 11 + \frac{19}{x-2} dx$$

$$\int 2x dx + \int 11 dx + \int \frac{19}{x-2} dx$$

$$\frac{2x^2}{2} + 11x + 19 \int \frac{1}{x-2} dx \quad \begin{array}{l} u = x-2 \\ du = dx \end{array}$$

$$19 \int \frac{1}{u} du$$

$$19 \ln|u| + C$$

$$x^2 + 11x + 19 \ln|x-2| + C$$

Ex. 6

$$\int \frac{2x}{(x+1)^2} dx$$

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$$\int \frac{2x}{(x+1)^2} dx$$

Stand and DeliverIntegral of Trig. Functions

5.2

$$\int \sin u \, du = -\cos u + c$$

$$\int \cos u \, du = \sin u + c$$

$$\int \tan u \, du = -\ln |\cos u| + c$$

$$\int \cot u \, du = \ln |\sin u| + c$$

$$\int \sec u \, du = \ln |\sec u + \tan u| + c$$

$$\int \csc u \, du = -\ln |\csc u + \cot u| + c$$

$$\int \tan u \, du = -\ln |\cos u| + c$$

$$\int \frac{\sin u}{\cos u} \, du$$

$$= \int \frac{1}{u} \, du$$

$$= \ln |u| + c$$

$$= \ln |\cos u| + c$$

$$u = \cos u$$

$$du = -\sin u \, du$$

$$-du = \sin u \, du$$

Ex. 7

$$\int \sec \frac{x}{2} dx$$

$$u = \frac{x}{2}$$

$$du = \frac{1}{2} dx$$

$$2 du = dx$$

$$2 \int \sec u du$$

$$2 \ln \left| \sec \frac{x}{2} + \tan \frac{x}{2} \right| + C$$

Ex. 8

$$\int_e^{e^2} \frac{1}{x \ln x} dx$$

$$\int_e^{e^2} \frac{1}{\ln x} \cdot \frac{1}{x} dx$$

$$\int_{\ln e}^{\ln e^2} \frac{1}{u} du$$

$$\int_1^2 \frac{1}{u} du$$

$$\ln|u| \Big|_1^2$$

$$F(2) - F(1)$$

$$\ln 2 - \ln 1$$

$$\ln 2 - 0$$

$$\ln 2$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$u = \ln x$$

$$\ln e^2$$

$$\ln e$$

$$\ln 2 - \ln 1$$

$$\ln \frac{2}{1} = \ln 2$$



Ex. 9 Find  $F'(x)$

$$F(x) = \int_1^{x^2} \frac{1}{t} dt$$

$$F'(x) = \frac{d}{dx} \int_1^{x^2} \frac{1}{t} dt$$

$$\frac{1}{x^2} \cdot 2x$$

$$= \frac{2}{x}$$

Ex. 10

Find the average value:

$$f(x) = \frac{4(x+1)}{x^2} \quad [2, 4]$$

$$\frac{1}{b-a} \int_a^b f(x) dx$$

$$\frac{1}{4-2} \int_2^4 \frac{4(x+1)}{x^2} dx$$

$$\frac{1}{2} \int_2^4 \frac{4(x+1)}{x^2} dx$$

$$2 \int_2^4 \frac{x+1}{x^2} dx$$

$$2 \left[ \int_2^4 \frac{x}{x^2} + \frac{1}{x^2} dx \right]$$

$$2 \left[ \int_2^4 \frac{1}{x} + x^{-2} dx \right]$$

$$2 \left[ \ln|x| + \frac{x^{-1}}{-1} \right]_2^4$$

$$\frac{x^{-1}}{-1} = -\frac{1}{x}$$

$$F(4) - F(2)$$

$$2 \left[ \left( \ln 4 - \frac{1}{4} \right) - \left( \ln 2 - \frac{1}{2} \right) \right]$$

$$2 \left[ \ln 4 - \ln 2 - \frac{1}{4} + \frac{1}{2} \right]$$

$$2 \left[ \ln \frac{4}{2} + \frac{1}{4} \right]$$

$$2 \left[ \ln 2 + \frac{1}{4} \right]$$

$$2 \ln 2 + \frac{1}{2}$$

$$\ln 2^2 + \frac{1}{2}$$

$$\ln 4 + \frac{1}{2}$$