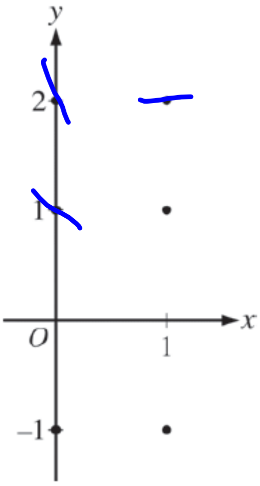
Chapter 6
Differential Equations

day 2 warm up after slope field day

2015 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

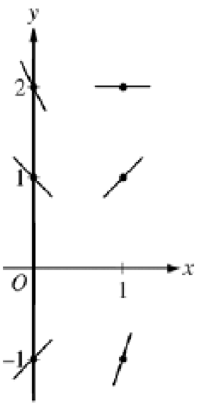


$(0,2)$ $2(0)-2 = -2$

$(0,1)$ $2(0)-1 = -1$

$(1,2)$ $2(1)-2 = 0$

(a)



$$2 : \begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$$

Vocab building...

derivative

differentiable

difference

differentiate

differential equation

Ex. 1

Solve the differential equation:

$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = x \cos x^2 \, dx$

$\int dy = \int x \cos x^2 \, dx$

$y = \int x \cos x^2 \, dx$

$\frac{1}{2} \int \cos u \, du$

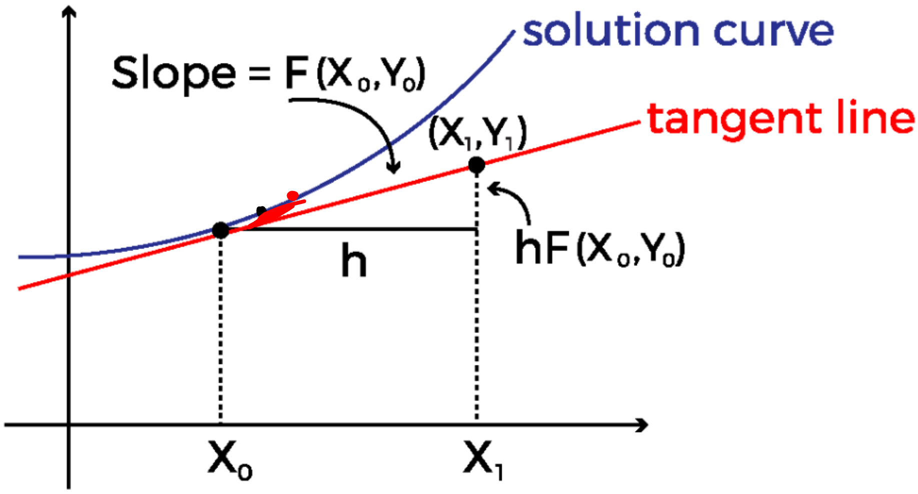
$\frac{1}{2} \sin x^2 + C$

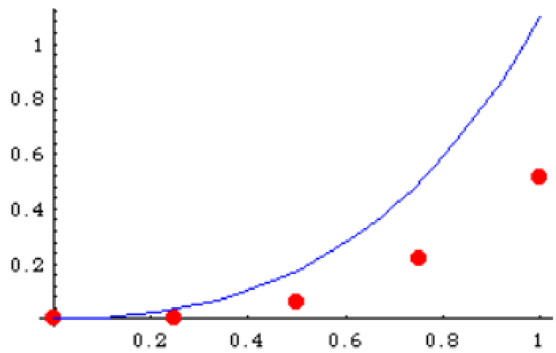
$u = x^2$

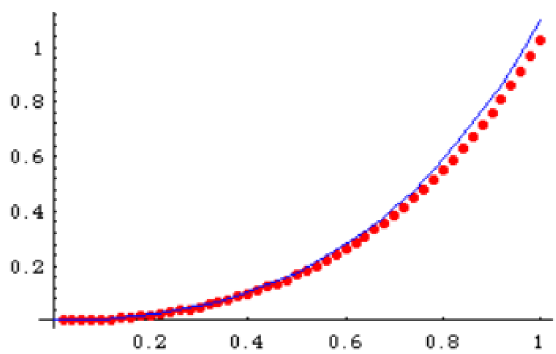
$du = 2x \, dx$

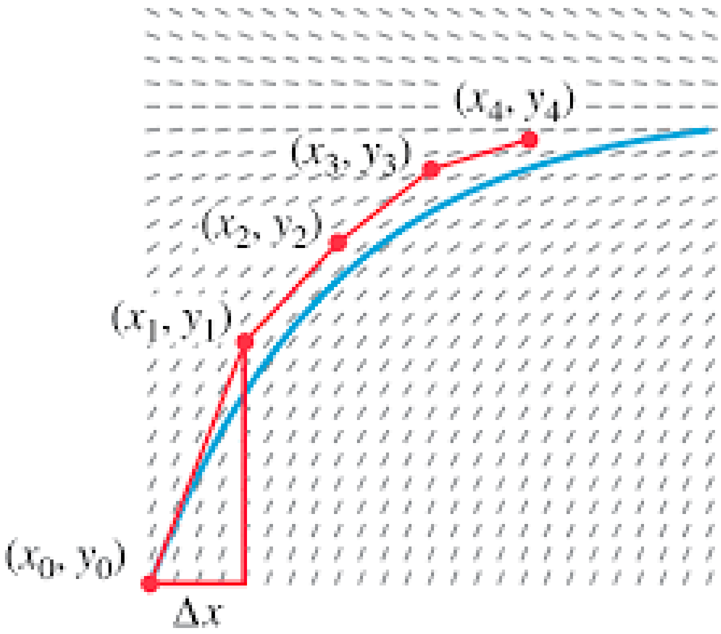
$\frac{1}{2} du = x \, dx$

Euler's Method, is just another technique used to analyze a Differential Equation, which uses the idea of local linearity or linear approximation, where we use small tangent lines over a short distance to approximate the solution to an initial-value problem.









Stand and Deliver

"Oiler"

Euler's Method

6.1

Table with $x,\ y,\ \frac{dy}{dx},\ dy = dx \cdot \frac{dy}{dx}$

- to be used when solving a differential equation
- the smaller the Δx (dx) the better the approximation

Ex. 2 $y' = x - y$ through $(0, 1)$

$\frac{dy}{dx} = x - y$ $\Delta x = .1$ $n = 3$

$dx = .1$

use to predict when $x = .3$

x	0	.1	.2	.3		
y	1	$y+dy$ $1-.1$ $.9$	$y+dy$ $.9-.08$ $.82$	$y+dy$ $.82-.062$ $.754$		
$x - y = \frac{dy}{dx}$	$0-1$ -1	$.1-.9$ $-.8$	$.2-.82$ $-.62$			
$dy = dx \cdot \frac{dy}{dx}$ $(.1)$	$-.1$	$-.08$	$-.062$			

$\Delta x = .1$
$dx = .1$

$$\frac{dy}{dx} = \frac{dy}{dx}$$

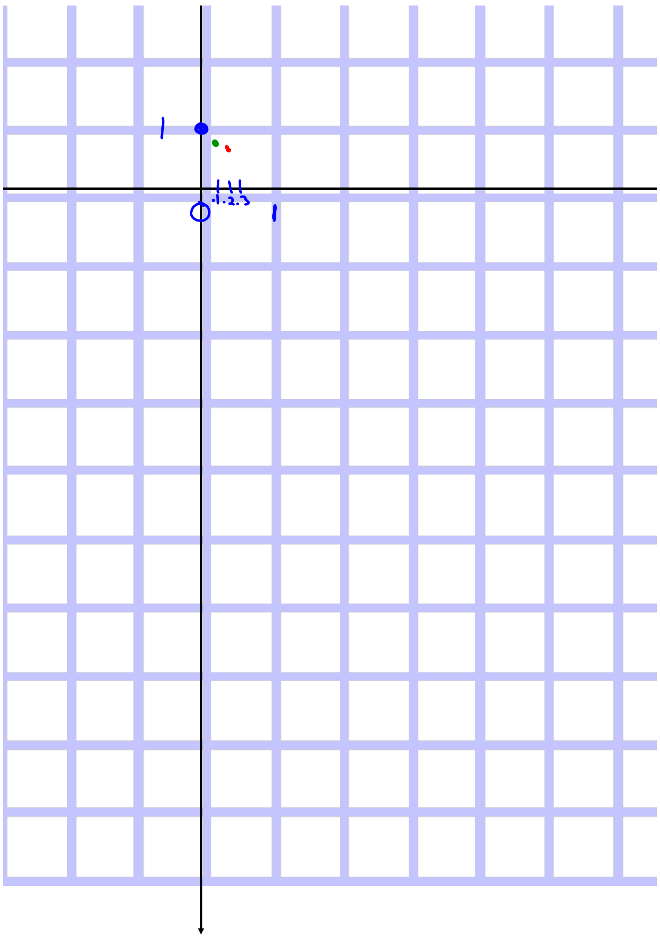
$$-1 = \frac{dy}{.1}$$

$$-.1 = dy$$

$$-.8 = \frac{dy}{.1}$$

$$-.08 = dy$$

$$-.62 = \frac{dy}{.1}$$



2009 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

$(-1, 2)$ $(0, \_\_)$

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

$\Delta x = .5 = \frac{1}{2}$

$dx = .5$

x	-1	$-.5$	0
y	2	$y + dy$ $2+2$ 4	$4 + \frac{1}{4}$ $4\frac{1}{4}$ or $\frac{17}{4}$ or 4.25
$6x^2 - x^2y = \frac{dy}{dx}$	$6(-1)^2 - (-1)^2(2)$ $6-2$ 4	$6(\frac{1}{2})^2 - (\frac{1}{2})^2 4$ $\frac{6}{4} - 1$ $\frac{3}{2} - 1 = \frac{1}{2}$	
$dx \cdot \frac{dy}{dx} = dy$ $(\frac{1}{2})(\ )$	$\frac{1}{2}(4)$ 2	$\frac{1}{2}(\frac{1}{2})$ $\frac{1}{4}$	

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Question 4

(a) $f\left(-\frac{1}{2}\right) \approx f(-1) + \left(\left.\frac{dy}{dx}\right|_{(-1,2)}\right) \cdot \Delta x$

$= 2 + 4 \cdot \frac{1}{2} = 4$

$f(0) \approx f\left(-\frac{1}{2}\right) + \left(\left.\frac{dy}{dx}\right|_{\left(-\frac{1}{2}, 4\right)}\right) \cdot \Delta x$

$\approx 4 + \frac{1}{2} \cdot \frac{1}{2} = \frac{17}{4}$

2 : { 1 : Euler's method with two steps
1 : answer