If, $F(x)=\int_0^x \sin^2(2t)\,dt$ then F'(x) =

$F'(x)=\frac{d}{dx}\left[\int_0^x \sin^2(2t)\,dt\right]$ $\sin^2(2x)\,(1)$

(A) $-\cos^2(2x)$ (B) $\cos^2(2x)$ (C) $\sin^2(2x)$

(D) $\frac{1}{2}\sin^2(2x)$ (E) $4\sin(2x)\cos(2x)$

5.1 The Natural Log Function: Differentiation

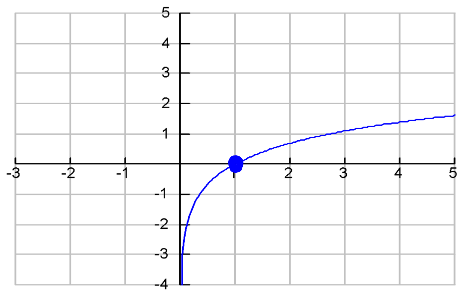
Objective: You will be able to:

- use properties of the ln function
- find derivatives of functions involving ln

Warm Up: How much do you remember about:

$$y = \ln x$$

$\ln = \log_? = e$



domain: $(0, \infty)$

range: $(-\infty, \infty)$

continuous
concave down

Properties of Logarithms:

1. $\ln 1 = x \longrightarrow 0$
   $e^x = 1 \quad x = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln(a/b) = \ln a - \ln b$

Expand:

$$\ln \sqrt[3]{\frac{x^3 - 2}{x^4}}$$

$$\ln\left(\frac{x^3-2}{x^4}\right)^{1/3}$$

$$\frac{1}{3}\left(\ln(x^3-2) - \ln x^4\right)$$

$$\frac{1}{3}\left(\ln(x^3-2) - 4\ln x\right)$$

Condense:
2 [ln x - ln(x + 1) - ln(x - 1)]

$$2\left[\ln \frac{x}{x+1} - \ln(x-1)\right]$$

$$2\left[\ln \frac{\frac{x}{x+1}}{x-1}\right]$$

$$2\left[\ln \frac{x}{(x+1)(x-1)}\right]$$

$$\ln\left(\frac{x}{x^2-1}\right)^2$$

Stand and Deliver

Derivative of Natural Log

5.1

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u', u > 0$$

$$= \frac{u'}{u}$$

u Represents an expression

Remember all the strategies of taking derivatives

- power
- product
- quotient
- chain
- implicit
- ln x

Ex. 1

$$\frac{d}{dx}[\ln 2x] =$$

$$\frac{1}{2x} \cdot 2$$

$$\frac{2}{2x} = \frac{1}{x}$$

$\ln 2 + \ln x$

$\frac{d}{dx}$

$\frac{1}{x}$

Ex. 2

$$\frac{d}{dx}[\ln (x^2 + 1)] =$$

$$y' = \frac{1}{x^2+1} \cdot 2x$$

$$y' = \frac{2x}{x^2+1}$$

Ex. 3

$$\frac{d}{dx}[(\ln x)^4] =$$

$$(\ln x)^4 \neq \ln x^4$$

$$4(\ln x)^3 \cdot \frac{1}{x}$$

$$\frac{4(\ln x)^3}{x}$$

Ex. 4

Find y'

$$y = \ln\sqrt{x^2 - 3}$$

$$y = \ln(x^2-3)^{1/2}$$

$$y = \frac{1}{2}\ln(x^2-3)$$

$$y' = \frac{1}{2}\left(\frac{1}{x^2-3}\cdot 2x\right)$$

$$y' = \frac{x}{x^2-3}$$

Ex. 4

Find y'

$$y = \ln\sqrt{x^2 - 3}$$

$$y' = \frac{1}{(x^2-3)^{1/2}} \cdot \frac{1}{\cancel{2}}(x^2-3)^{-1/2} \cdot \cancel{2}x$$

$$\frac{x}{(x^2-3)^1}$$

Ex. 5

Find y'

$$y = \ln \frac{x(x^2+1)}{\sqrt{2x^3-3}}$$

$$y = \ln x + \ln(x^2+1) - \frac{1}{2}\ln(2x^3-3)$$

$$y' = \frac{1}{x} + \frac{1}{x^2+1}\cdot 2x - \frac{1}{2}\cdot\frac{1}{(2x^3-3)}\cdot 6x^2$$

$$y' = \frac{1}{x} + \frac{2x}{x^2+1} - \frac{3x^2}{2x^3-3}$$

Ex. 6

Find y'

before working...see next slide for absolute value

y = ln |cos x|

$$y' = \frac{1}{\cos x}\cdot -\sin x$$

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

Because the natural logarithm is undefined for negative numbers, you will often encounter expressions of the form $\ln|u|$. The following theorem states that you can differentiate functions of the form $y = \ln|u|$ as if the absolute value notation was not present.

THEOREM 5.4 DERIVATIVE INVOLVING ABSOLUTE VALUE

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{u'}{u}.$$

PROOF If $u > 0$, then $|u| = u$, and the result follows from Theorem 5.3. If $u < 0$, then $|u| = -u$, and you have

$$\begin{aligned}\frac{d}{dx}[\ln|u|] &= \frac{d}{dx}[\ln(-u)] \\ &= \frac{-u'}{-u} \\ &= \frac{u'}{u}.\end{aligned}$$

Ex. 7 Find $f'(x)$

$$f(x) = \ln\left(\frac{2x}{x+3}\right)$$

$$y = \ln(2x) - \ln(x+3)$$

$$y' = \frac{1}{2x}\cdot 2 - \frac{1}{x+3}\cdot 1$$

$$y' = \frac{1}{x} - \frac{1}{x+3}$$