5.7 Integrals of Inverse Trigonometric Functions

Objective: You will be able to:

- integrate inverse trig functions

Stand and Deliver

5.7

Integrals of Inverse Trig. Functions

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

Ex. 1

$$\int \frac{3}{\sqrt{1-4x^2}}\,dx$$

$$3\int \frac{1}{\sqrt{1-4x^2}}\,dx$$

$\arcsin \frac{u}{a} + C$

$u = 1-4x^2$

$du = -8x\,dx$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

$a^2 = 1$

$a = 1$

$u^2 = 4x^2$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$3 \cdot \frac{1}{2}\arcsin\frac{2x}{1} + C$$

$$\frac{3}{2}\arcsin 2x + C$$

Ex. 2

$$\int \frac{1}{4+(x-1)^2}\,dx$$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

$a^2 = 4$

$a = 2$

$u^2 = (x-1)^2$

$u = x-1$

$du = dx$

$$\frac{1}{2}\arctan\frac{x-1}{2} + C$$

Ex. 3

$$\int \frac{x^4-1}{x^2+1}\,dx$$

$$\int \frac{(x^2+1)(x^2-1)}{(x^2+1)}\,dx$$

$$\int x^2-1\,dx$$

$$\frac{x^3}{3}-x+C$$

Ex. 4

$$\int \frac{1}{x\sqrt{x^4-4}}\,dx$$

$$\int \frac{x}{x\sqrt{x^4-4}}\,dx$$

$$\int \frac{x}{x^2\sqrt{x^4-4}}\,dx$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+C$$

$u^2=x^4$ $\quad a^2=4$ $\quad a=2$

$u=x^2$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$\frac{1}{2}\cdot\frac{1}{2}\operatorname{arcsec}\frac{|x^2|}{2}+C$

$$\frac{1}{4}\operatorname{arcsec}\frac{x^2}{2}+C$$

Ex. 5

$$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+C$

$a^2=1$ $\quad u^2=\sin^2 x$

$a=1$ $\quad u=\sin x$

$du=\cos x\,dx$

$\frac{1}{1}\arctan\frac{\sin x}{1}\Big]_0^{\pi/2}$

$\arctan \sin x\Big]_0^{\pi/2}$

$F(\pi/2) - F(0)$

$\arctan \sin(\pi/2) - \arctan \sin 0$

$\arctan(1) - \arctan 0$

$\pi/4 - 0$

$\frac{\pi}{4}$

Ex. 6

review completing the square

$$\int \frac{1}{x^2+4x+5}\,dx$$

$$\int \frac{1}{x^2+4x+5-1+1}\,dx$$

$$\int \frac{1}{x^2+4x+4+1}\,dx$$

$$\int \frac{1}{(x+2)^2+1}\,dx$$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+C$

$a^2=1$ $\quad u^2=(x+2)^2$

$a=1$ $\quad u=(x+2)$

$du=dx$

$\frac{1}{1}\arctan\frac{x+2}{1}+C$

$\arctan(x+2)+C$

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17. $\int \frac{1}{x^2+4x+5}\,dx =$

(A) $\arctan(x+2)+C$

(B) $\arcsin(x+2)+C$

(C) $\ln\left|x^2+4x+5\right|+C$

(D) $\frac{1}{\frac{1}{3}x^3+2x^2+5x}+C$