5.6 Derivatives of Inverse Trigonometric Functions

Objective: You will be able to:

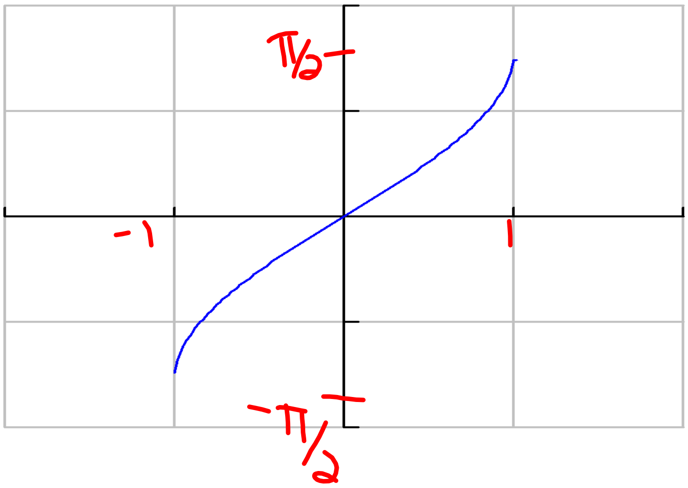
- develop properties of inverse trig functions
- differentiate inverse trig functions

Warm Up

Function	*Domain*	*Range*
arcsin $\sin^{-1}$		
arccos $\cos^{-1}$		
arctan $\tan^{-1}$		

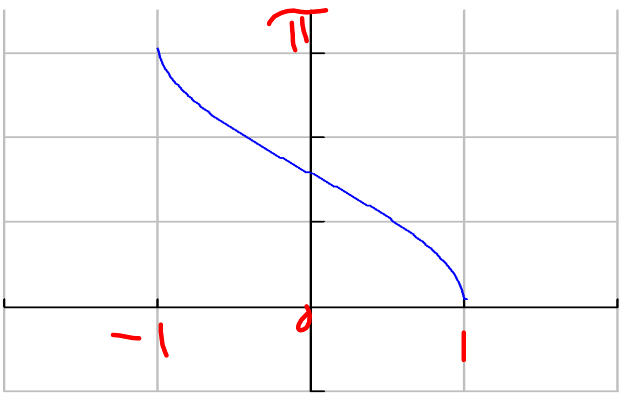
arcsin

$\sin^{-1}$

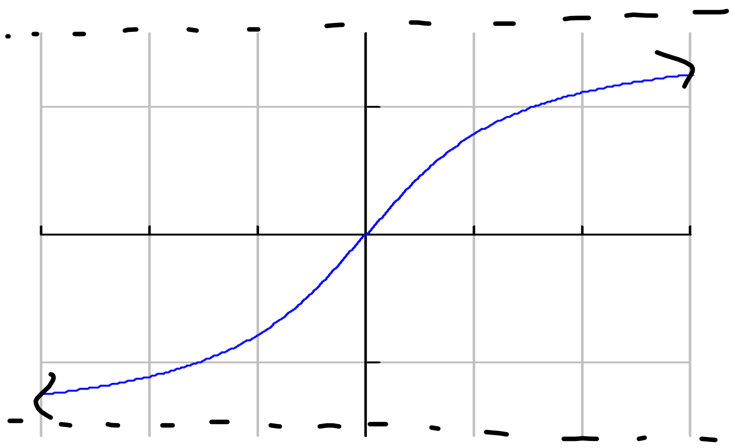


arccos

$\cos^{-1}$



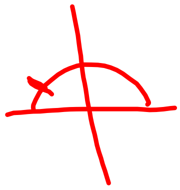
arctan
$\tan^{-1}$



Function	*Domain*	*Range*
arcsin $\sin^{-1}$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
arccos $\cos^{-1}$	$-1 \le x \le 1$	$0 \le y \le \pi$
arctan $\tan^{-1}$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Ex. 1 Precalc/Trig Review

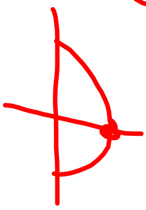
means "angle whose sine is"



a. arcsin 0

$\sin\theta = 0$

0



b. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$5\pi/6$

Ex. 2

with calculator in radians

arcsin (- .39) =

Ex. 3

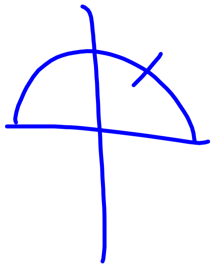
Hint: draw △
label what you know

$$a.\ \tan\left(\arccos\frac{\sqrt{2}}{2}\right)$$

$$\left(\cos\theta = \frac{\sqrt{2}}{2}\right)$$

$$\tan\left(\pi/4\right)$$

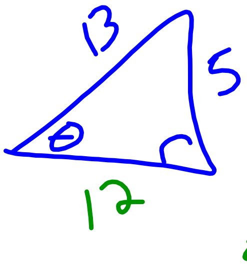
$$1$$



$$b.\ \cos\left(\arcsin\frac{5}{13}\right)$$

$\frac{opp}{hyp}$ $\sin\theta = \frac{5}{13}$

$$\frac{12}{13}$$

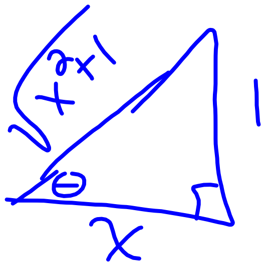


$5^2+b^2=13^2$

$c. \cos(\operatorname{arc}\cot x)$

$S\frac{o}{h}\ C\frac{a}{h}\ T\frac{o}{a}$

$\frac{adj}{opp}\ \left(\cot\theta = \frac{x}{1}\right)$



$x^2 + 1^2 = c^2$

$\sqrt{x^2+1} = \sqrt{c^2}$

$\sqrt{x^2+1} \neq x+1$

$\sqrt{16+1} \neq 4+1$

$$\frac{x}{\sqrt{x^2+1}}$$

Now let's use what we know to come up with the derivative of arcsin

$y = \sin^{-1}(x)$

a. rewrite: $\sin y = \sin(\sin^{-1}(x))$

$\sin y = x$

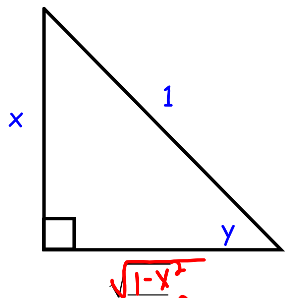
or... $x = \sin y$

b. find the derivative (implicit!!) $1 = \cos y \frac{dy}{dx}$

c. solve for $\frac{dy}{dx}$ $\quad \frac{1}{\cos y} = \frac{dy}{dx}$

d. use right Δ trig to subs. cos y $\quad \frac{dy}{dx} = \frac{1}{}$

e. the derivative is... $\quad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$



$x^2 + b^2 = 1^2$

$\sqrt{b^2} = \sqrt{1-x^2}$

Now you find the derivative of arccos x

$$y = \cos^{-1}(x)$$

Stand and Deliver

5.6

Derivative of Inverse Trig. Functions

$$\frac{d}{du}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{du}\arctan u = \frac{u'}{1+u^2}$$

$$\frac{d}{du}\arccos u = \frac{-u'}{\sqrt{1-u^2}} \qquad \frac{d}{du}\operatorname{arccot} u = \frac{-u'}{1+u^2}$$

$$\frac{d}{du}\operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{du}\operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Ex. 4

Find the derivative...

a. arcsin t^2

$\frac{d}{du} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$

$$\frac{2t}{\sqrt{1-(t^2)^2}}$$

$$\frac{2t}{\sqrt{1-t^4}}$$

Ex. 4

Find the derivative...

b. x^2 arctan x

product

$\frac{d}{du} \arctan u = \frac{u'}{1+u^2}$

$$x^2 \cdot \frac{1}{1+x^2} + \arctan x\,(2x)$$

$$\frac{x^2}{1+x^2} + 2x \arctan x$$