5.5 Bases Other than e

Objective: You will be able to:

- define exponential functions that have bases other than e
- differentiate and integrate exponential functions
- model compound interest and exponential growth

Change of Base

The change of base formula states that $\log_a x$ can be converted to different bases.

Base b $\log_a x = \dfrac{\log_b x}{\log_b a}$

***Base 10 $\log_a x = \dfrac{\log x}{\log a}$

***Base *e* $\log_a x = \dfrac{\ln x}{\ln a}$

Review of logs:

Ex. 1

a) evaluate without a calculator

$\log_{27} 9 = x$

$27^x = 9$

$(3^3)^x = 3^2$

$3^{3x} = 3^2$

$3x = 2$

$x = 2/3$

b) write as a log

$16^{3/4} = 8$

$\log_{16} 8 = 3/4$

c) write as an exponent

$\log_3 \frac{1}{9} = -2$

$3^{-2} = \frac{1}{9}$

d) solve

$\log_3 9 = x$

$3^x = 9$

$$x^2 - x = \log_3 9$$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

e) solve

$A = P(1 + \frac{r}{n})^{nt}$

$$(1 + \frac{.10}{365})^{365t} = 2$$

$\log_{(1+\frac{.10}{365})} (1+\frac{.10}{365})^{365t} = \log_{(1+\frac{.10}{365})} 2$

$365t =$

f) solve

$$\log_5 \sqrt{x-4} = 3.2$$

$$\log_5 (x-4)^{\frac{1}{2}} = 3.2$$

$$\frac{1}{2}\log_5 (x-4) = 3.2$$

$$\log_5 (x-4) = 6.4$$

$$5^{6.4} = x-4$$

$$5^{6.4} + 4 = x$$

Stand and Deliver

5.5

Derivative of a^u

$$\frac{d}{dx}[a^u] = (\ln a)a^u \cdot u'$$

Stand and Deliver

5.5

Derivative of $\log_a u$

$$\frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \cdot u'$$

Ex. 2

Find the derivative:

$\frac{d}{dx}\{a^u\} = \ln a \cdot a^u \cdot u'$

a) $y = 3^{x-4}$

$y' = \ln 3 \cdot 3^{x-4}(1)$

$y' = \ln 3 \cdot 3^{x-4}$

Ex. 2

Find the derivative:

b) $y = x(6^{-2x})$

product Rule

$\frac{d}{dx}[a^u]$

$y' = x \cdot \ln 6 (6^{-2x})(-2) + 6^{-2x}(1)$

$y' = -2x(\ln 6)6^{-2x} + 6^{-2x}$

$= \ln 6(-2x)6^{-2x}$

Ex. 2

Find the derivative:

$\frac{d}{dx}[\log_a u] = \frac{1}{\ln a \cdot u} \cdot u'$

c) $y = \log 2x$

$y = \log_{10} 2x$

$y' = \frac{1}{\ln 10(2x)} \cdot 2$

$y' = \frac{2}{\ln 10(2x)}$

Ex. 2

Find the derivative:

d) $y = \log \frac{x^2 - 1}{x}$

$\log(x^2-1) - \log x$

$\frac{1}{\ln 10 (x^2-1)} \cdot 2x - \frac{1}{(\ln 10)x} \cdot 1$

$\frac{2x}{\ln 10 (x^2-1)} - \frac{1}{\ln 10 (x)}$

Stand and Deliver

5.5

Integral of a^u

$$\int a^u du = \frac{1}{\ln a} \cdot a^u + c$$