5.4 Exponential Functions

Objective: You will be able to:

- develop properties of exponential functions
- differentiate e^x
- integrate e^x

Warm Up

chart inverse from 5.3

Given to the right is a table of values for f, g, f', and g'. Use the values in the table to find each indicated value in the boxes below.

x	f	g	f'	g'
−2	1	2	0	3
0	−4	−3	−1	2
1	3	−2	2	1
3	1	1	−3	−2

Find the value of

$$(f^{-1})'(3)$$

$f^{-1}(3) = ?$ $(3, \underline{1})$

$f(x) = 3$ $(\underline{1}, 3)$

$$\frac{f'(1) = 2}{f^{-1}(3) = 1/2}$$

Find the value of

$$(g^{-1})'(2)$$

$g^{-1}(2) = \dfrac{}{(2, -2)}$

$g(x) = 2$

$(-2, 2)$

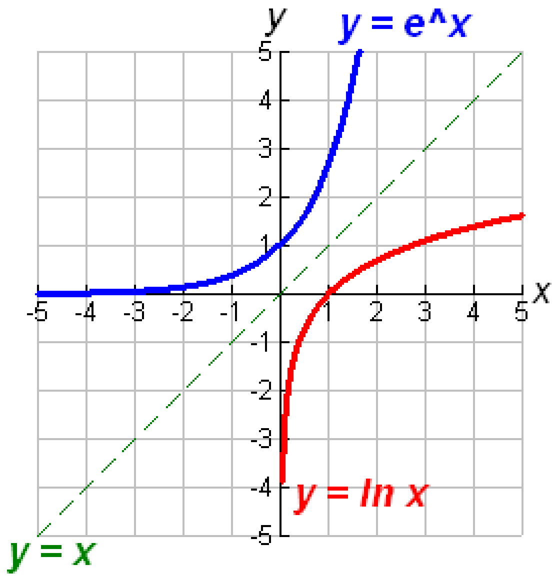
$g'(-2) = 3$

$g^{-1}(2) = 1/3$

Exponential Form $y = e^x$

Logarithmic Form $\ln y = x$

e and ln are inverses



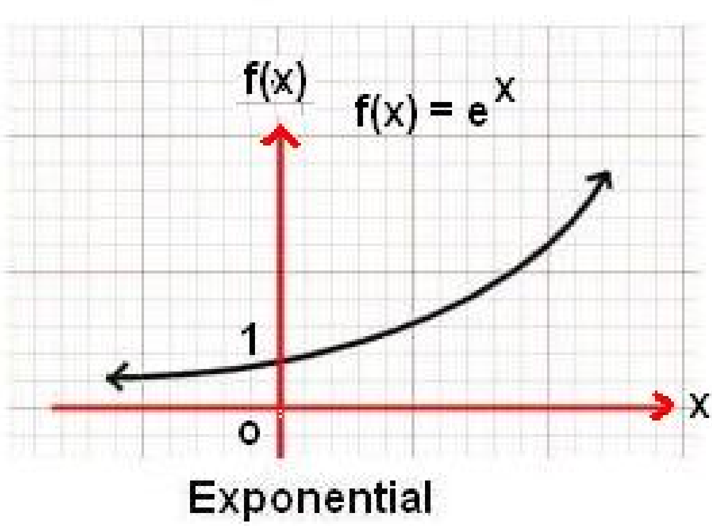
properties of exponents

$$e^a e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

domain $(-\infty, \infty)$

range $(0, \infty)$



Ex. 1

a) write in log form:

$$e^{-2} = .1353$$

$$\log_e .1353 = -2$$

$$\ln(.1353) = -2$$

b) write in exponential form:

$$\ln 0.5 = -0.6931$$

$$e^{-.6931} = .5$$

Ex. 2

a) solve

$$-6 + 3e^{2x} = 9$$
$$+6 \qquad\qquad +6$$
$$\frac{3e^{2x}}{3} = \frac{15}{3}$$
$$e^{2x} = 5$$
$$\ln e^{2x} = \ln 5$$
$$\frac{2x}{2} = \frac{\ln 5}{2}$$
$$x = \frac{\ln 5}{2}$$

b) solve $\ln_e(4x) = 12$

$$\frac{e^{12}}{4} = \frac{\cancel{4}x}{\cancel{4}}$$

$$\ln(4x) = 12$$
$$e^{\ln(4x)} = e^{12}$$
$$\frac{\cancel{4}x}{\cancel{4}} = \frac{e^{12}}{4}$$

Stand and Deliver

Derivative of e

5.4

$$\frac{d}{dx}[e^{x}] = e^{x}$$

$$\frac{d}{dx}[e^{u}] = e^{u} \cdot u'$$

Ex. 3

Find the derivative:

product Rule

a) $y = e^{-2x}$

$y' = e^{-2x}(-2)$

$y' = -2e^{-2x}$

b) $y = x^2e^{-x}$

$y' = x^2 \cdot e^{-x}(-1) + e^{-x}(2x)$

$y' = -x^2e^{-x} + 2xe^{-x}$

Ex. 3

Find the derivative:

c) $y = e^3 \ln x$

$$y' = e^3 \cdot \frac{1}{x}$$

$$y' = \frac{e^3}{x}$$

Ex. 3

Find the derivative:

d) $e^{xy} + x^2 - y^2 = 10$

$$e^{xy}\cdot\left(x\frac{dy}{dx}+y(1)\right)+2x-2y\frac{dy}{dx}=0$$

$$e^{xy}\left(x\frac{dy}{dx}+y\right)+2x-2y\frac{dy}{dx}=0$$

$$e^{xy}x\frac{dy}{dx}+e^{xy}\cdot y+2x-2y\frac{dy}{dx}=0$$

$$e^{xy}x\frac{dy}{dx}-2y\frac{dy}{dx}=-ye^{xy}-2x$$

$$\frac{dy}{dx}\frac{\cancel{(xe^{xy}-2y)}}{\cancel{xe^{xy}-2y}}=\frac{-ye^{xy}-2x}{xe^{xy}-2y}$$

Stand and Delivér

Integral of e 5.4

$$\int e^x dx = e^x + c$$

$$\int e^u du = e^u + c$$

Ex. 4

$u = -x^4$

$du = -4x^3\,dx$

a) $\int e^{-x^4}(-4x^3)dx$

$\int e^u\,du$

$e^u + c$

$e^{-x^4} + c$

Ex. 4

b) $\int x^2 e^{\left(\frac{x^3}{2}\right)} dx$

$u = \frac{x^3}{2}$

$du = \frac{3x^2}{2} dx$

$\frac{2}{3} du = x^2 dx$

$\frac{2}{3} \int e^u du$

$\frac{2}{3} e^u + c$

$\frac{2}{3} e^{x^3/2} + c$

Ex. 4

c) $\int \frac{e^{1/x^2}}{x^3}dx$

$\int e^{1/x^2} \cdot \frac{1}{x^3} dx$

$u = \frac{1}{x^2} = x^{-2}$

$du = -2x^{-3} dx$

$du = -2 \cdot \frac{1}{x^3} dx$

$-\frac{1}{2} du = \frac{1}{x^3} dx$

$-\frac{1}{2}\int e^u \, du$

$-\frac{1}{2} e^u + C$

$-\frac{1}{2} e^{1/x^2} + C$

Chpt. 5 Stand + Deliver

- $\frac{d}{dx} \ln u$
- $\int \frac{1}{u} dx$
- 6 Trig Functions
- $\frac{d}{dx} e^u$
- $\int e^u dx$