4.5 Integration by U-Substitution

Objective: You will be able to:

- recognize and integrate composite functions using substitution

Warm Up

Find the derivative of $y = (2x^2 + 3)^3$

Chain Rule . . .

$$\int f(u)du = F(u) + C$$

Ex. 1

$$\int (x^2 - 9)^3 2x\,dx$$

$u = x^2 - 9$

$\frac{du}{dx} = 2x \cdot dx$

$du = 2x\,dx$

$$\int u^3\,du$$

$$\frac{u^4}{4} + C$$

$$\frac{(x^2-9)^4}{4} + C$$

Ex. 2

$$\int x^2\sqrt{x^3+2}\,dx$$

$u = x^3+2$

$dx \cdot \frac{du}{dx} = 3x^2\,dx$

differential → $\frac{du}{3} = \frac{3x^2\,dx}{3}$

$\frac{1}{3}du = x^2dx$

$\int \sqrt{x^3+2}\,x^2\,dx$

$\int \sqrt{u}\,\frac{1}{3}\,du$

$\frac{1}{3}\int u^{1/2}\,du$

$\frac{1}{3}\left(\frac{2}{3}\cdot u^{\frac{3}{2}} + C\right)$

$\frac{2}{9}(x^3+2)^{3/2}+C$

Ex. 3

$$\int \frac{x^2}{(16-x^3)^2}dx$$

$u = 16-x^3$

$\frac{du}{-3} = \frac{-3x^2\,dx}{-3}$

$-\frac{1}{3}du = x^2dx$

$\int \frac{1}{(16-x^3)^2}\cdot x^2\,dx$

$-\frac{1}{3}\int \frac{1}{u^2}\,du$

$-\frac{1}{3}\int u^{-2}\,du$

$\left(-\frac{1}{3}\right)\cdot\frac{u^{-1}}{-1}+C$

$\frac{1}{3u}+C$

$\frac{1}{3(16-x^3)}+C$

Ex. 4

Solve: $\frac{dy}{dx} = \frac{10x^2}{\sqrt{1+x^3}}$

$dy = \frac{10x^2}{\sqrt{1+x^3}}\,dx$

$\int dy = \int \frac{10x^2}{\sqrt{1+x^3}}\,dx$

$y = \int \frac{10x^2}{\sqrt{1+x^3}}\,dx$

$u = 1+x^3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2dx$

$y = 10\int \frac{1}{\sqrt{1+x^3}} \cdot x^2\,dx$

$y = \frac{10}{3}\int \frac{1}{\sqrt{u}}\,du$

$y = \frac{10}{3}\int u^{-1/2}\,du$

$\frac{10}{3}\cdot 2 \cdot u^{1/2} + C$

$\frac{20}{3}(1+x^3)^{1/2} + C$

Ex. 5

$\int x\sin(x^2)dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \sin(x^2)\,x\,dx$

$\frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos u + C$

$-\frac{1}{2}\cos x^2 + C$

Ex. 6

$$\int x\sqrt{x+3}\,dx$$

$u = x+3$ → $x = u-3$

$du = dx$

$$\int x\sqrt{u}\,du$$

$$\int (u-3)\sqrt{u}\,du$$

$$\int (u-3)u^{1/2}\,du$$

$$\int u^{3/2} - 3u^{1/2}\,du$$

$$\frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C$$

$$\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$$

Ex. 7

$$\int \sec^2 x\,(\tan x + 3)\,dx$$

$u = \tan x + 3$

$du = \sec^2 x\,dx$

$$\int (\tan x + 3)\sec^2 x\,dx$$

$$\int u\,du$$

$$\frac{u^2}{2} + C$$

$$\frac{(\tan x + 3)^2}{2} + C$$