Stand and Deliver

Mean Value Theorem for Integrals 4.4

$$\int_a^b f(x)dx = f(c)(b-a)$$

average height width

Calculus = Geometry

Area Area

(area under curve) (rectangle)

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picture to go with the card

Theorems.gsp

go to tab mvt for integrals

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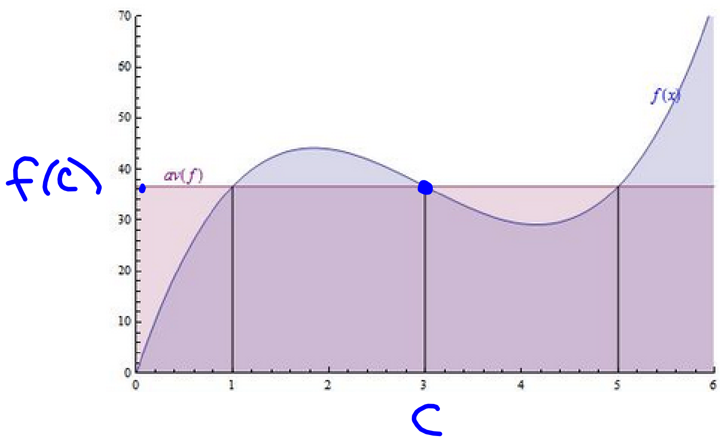
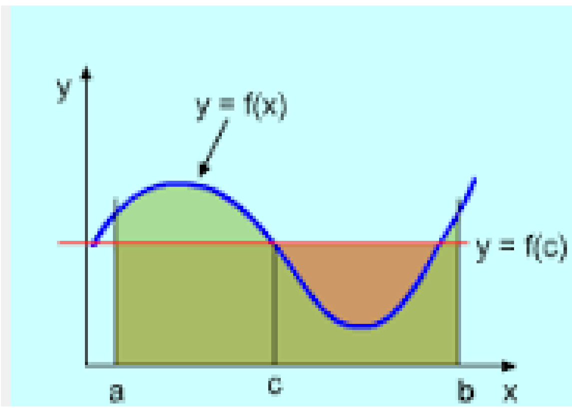
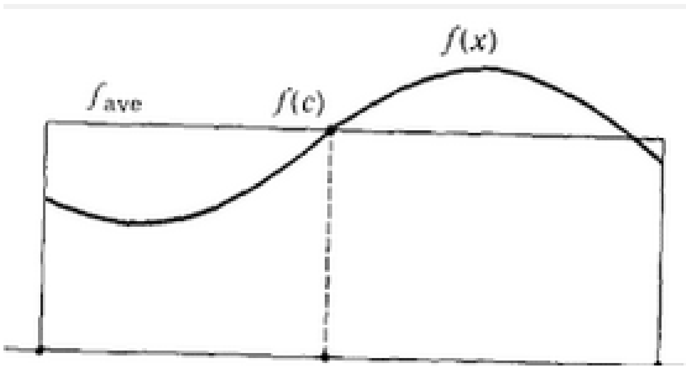
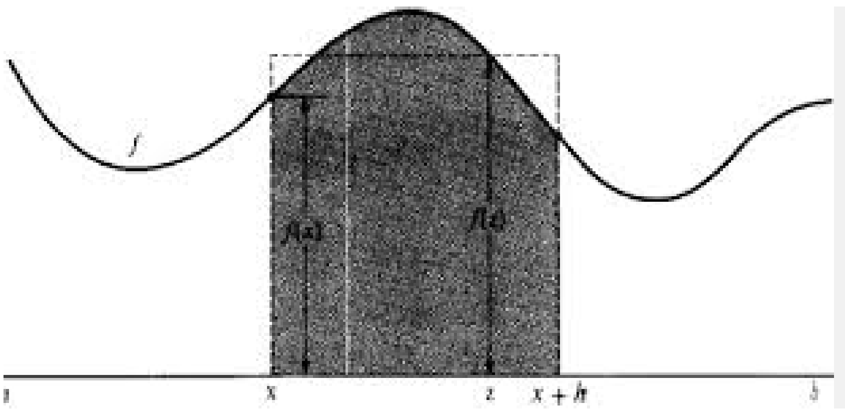

f(c)
av(f)
f(x)
c
0
10
20
30
40
50
60
70
0
1
2
3
4
5
6


y
y = f(x)
y = f(c)
a
c
b
x


f(x)
f ave
f(c)


f
x
z
x + h

To solve for the average value (average height) we can divide both sides by (b-a)

$$\frac{\int_a^b f(x)dx}{(b-a)} = \frac{\overset{\text{average height}}{f(c)}\,\overset{\text{width}}{\cancel{(b-a)}}}{\cancel{(b-a)}}$$

$$\frac{1}{b-a}\int_a^b f(x)dx = f(c)$$

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Calculus the Musical: Physics Extravaganza

Average Value (Average Height) 4.4

$$f(c) = \frac{1}{(b-a)}\int_a^b f(x)dx$$

average height

Ex. 6

Find c (specific x) guaranteed by the Mean Value Theorem

$f(x)=\frac{9}{x^3}$ on interval $[a,b]$ [1, 3]

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$

$$\frac{1}{3-1}\int_1^3 \frac{9}{x^3}\,dx = f(c)$$

$$\frac{9}{2}\int_1^3 x^{-3}\,dx = f(c)$$

$$\frac{9}{2}\left[\frac{x^{-2}}{-2}\right]_1^3$$

$$\frac{9}{-4}\left[\frac{1}{x^2}\right]_1^3$$

$$-\frac{9}{4}\left[\overbrace{\frac{1}{3^2}-\frac{1}{1^2}}^{F(3)-F(1)}\right]$$

$$-\frac{9}{4}\left[\frac{1}{9}-1\right]$$

$$-\frac{9}{4}\left[-\frac{8}{9}\right]$$

$$f(c)=2$$

$$f(x)=\frac{9}{x^3}$$

$$2=\frac{9}{x^3}$$

$$2x^3=9$$

$$x^3=9/2$$

$$x=\sqrt[3]{9/2}$$

Ex. 7 Find the average value $f(c)=?$

$$f(x) = \cos x \ , \quad [0, \pi/2]$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$

$$\frac{1}{\pi/2-0}\int_0^{\pi/2} \cos x\,dx = f(c)$$

$$\frac{2}{\pi}\left[\sin x\right]_0^{\pi/2}$$

$$\frac{2}{\pi}\left[\overbrace{\sin \pi/2 - \sin 0}^{f(\pi/2)-f(0)}\right]$$

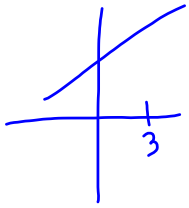
$$\frac{2}{\pi}\left[1-0\right]$$

$$2/\pi$$

Let's Discover the Fundamental Theorem of Calculus Part II

Find F(x) then find F'(x)

$$F(x) = \int_3^x (t + 2)\,dt$$



$$\left.\frac{t^2}{2} + 2t\right]_3^x$$

$$F(x) - F(3)$$

$$F(x) = \left(\frac{x^2}{2} + 2x\right) - \left(\frac{9}{2} + 6\right)$$

$$\frac{9}{2} + \frac{12}{2}$$

$$\frac{21}{2}$$

$$F(x) = \frac{x^2}{2} + 2x - \frac{21}{2}$$

$$F'(x) = \frac{2x}{2} + 2$$

$$F'(x) = x + 2$$

Find F(x) then find F'(x)

$$F(x) = \int_0^{x^3} \sin t\,dt$$

$$-\left[\cos t\right]_0^{x^3}$$

$$F(x^3) - F(0)$$

$$-\left[\cos x^3 - \cos 0\right]$$

$$-\left[\cos x^3 - 1\right]$$

$$F(x) = -\cos x^3 + 1$$

$$F'(x) = \sin x^3 \cdot 3x^2$$

$$F'(x) = 3x^2 \sin x^3$$

Stand and Deliver

4.4

Fundamental Theorem of Calculus Part II

Derivative of an Integral

$$\frac{d}{dx}\left[\int_a^{x} f(t)\,dt\right] = f(x)x'$$

?

$$\frac{d}{dx}\left[\int_a^{x} f(t)\,dt\right]$$

$$\frac{d}{dx}\Big[\ F(t)\ \Big]_a^{x}$$

$$\frac{d}{dx}\Big[F(x) - F(a)\Big]$$

$$\frac{d}{dx}F(x) - \frac{d}{dx}F(a)$$

$$F'(x) - 0$$ (derivative of f ' may include a chain rule)

$$f(x)$$

Let's go back at look at those 2 examples

Try 2 more, and let's use the theorem

$$\frac{d}{dx}\int_0^{x}(t\cos t)\,dt$$

$x\cos x\,(1)$

$x\cos x$

$$\frac{d}{dx}\int_{\pi}^{x^3}\sin t^2\,dt$$

$\sin(x^3)^2\cdot 3x^2$

$3x^2\sin x^6$

Wh

1. $\int \frac{3x - 2x^2}{\sqrt{x}}$

2. $\int \frac{\cos x}{1 - \cos^2 x}\, dx$

3. Solve the differential equation: $f''(x) = x^2$, $f'(0) = 1$, $f(0) = 6$

4. A ball is thrown vertically upward from the ground with an initial velocity of 45 ft/sec. How high will the ball go? Use $a(t) = -32$ ft/sec^2

5. Use your calculator to evaluate the given sum: $\sum_{x=1}^{8} (x^2 + 3x - 1)$

6. Given: $\int_{-1}^{7} f(x)\, dx = 12$ $\int_{2}^{7} f(x)\, dx = -4$ Find: $\int_{-1}^{2} f(x)\, dx =$

Attachments

Theorems.gsp