4.1 Antiderivatives and Indefinite Integrals

Objective: You will be able to:

- use indefinite integral notation for antiderivatives
- use basic integration rules to find:
- general solutions
- particular solutions

Warm Up

$$y' = 4x$$

What do think the original function looked like?

$$y = 2x^2$$

$$y = 2x^2 + 10$$

$$y = 2x^2 - 1{,}000{,}000$$

These solutions are called:

Find the antiderivative of:
(integral)

$f(x) = 3x^2 + 1$ $g(x) = \cos x$

$F(x) = x^3 + x + C$ $G(x) = \sin x + C$

These are called "general solutions" because...

"c" is called... constant of integration

Notation:

introduce integral sign

$$\frac{dy}{dx} = f(x)$$

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = f(x) \cdot dx$$

$$\int dy = \int f(x)\,dx$$

$$y = \int f(x)\,dx$$

Stand and Deliver

4.1

Power Rule to Integrate

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\quad n \neq -1$$

add one to the exponent,
divide by new exponent

To evaluate integrals:

1. Simplify / Rewrite
2. Integrate
3. Simplify

$\int (x^3 - 4x + 2)\,dx$

$\int x^3 - 4x + 2\,dx$

$$\frac{x^4}{4} - \frac{4x^2}{2} + 2x + C$$

$$F(x) = \frac{x^4}{4} - 2x^2 + 2x + C$$

$$\int \frac{x^2 + 2x - 3}{\sqrt{x}}\, dx$$

There is no reverse for the quotient rule.

$$\int \frac{x^2}{x^{1/2}} + \frac{2x}{x^{1/2}} - \frac{3}{x^{1/2}}\, dx$$

$$\int x^{3/2} + 2x^{1/2} - 3x^{-1/2}\, dx$$

$$\frac{x^{5/2}}{\frac{5}{2}} + \frac{2x^{3/2}}{3/2} - \frac{3x^{1/2}}{1/2} + C$$

$$\frac{2}{5}x^{5/2} + \frac{2}{3}\cdot 2x^{3/2} - 2(3)x^{1/2} + C$$

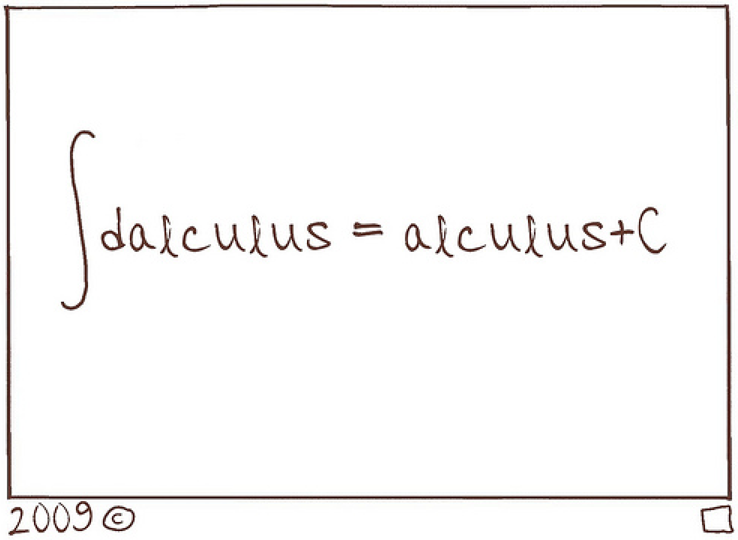
$$= \frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} - 6x^{1/2} + C$$

$$\int \left(\frac{1}{x} + 6\right) dx \qquad \int \frac{1}{x} + 6dx$$

$$\int x^{-1} + 6\, dx$$

Power Rule doesn't work

will learn in Chpt. 5



write 6 trig functions and deriv.

Stand and Deliver

Integration of Trig Functions

4.1

$\int \sin x\, dx = -\cos x + C$

$\int \sec^2 x\, dx = \tan x + C$

$\int \cos x\, dx = \sin x + C$

$\int \csc^2 x\, dx = -\cot x + C$

$\int \sec x \tan x\, dx = \sec x + C$

$\int \csc x \cot x\, dx = -\csc x + C$