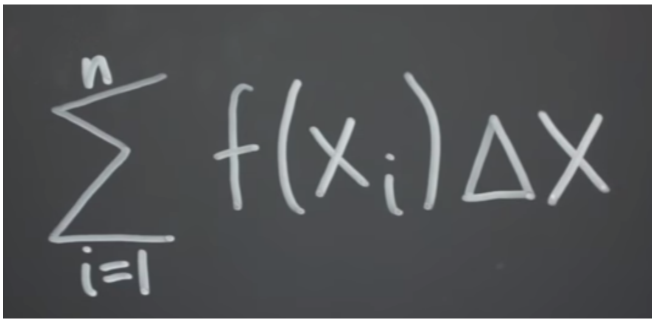
4.3 Reimann Sums
and Definite Integrals

Objective: You will be able to:
- understand the definition of a Reimann sum
- evaluate definite integrals

What do you think this means?

$$\lim_{\Delta x \to 0} \sum_{i=1}^{\infty} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

https://www.youtube.com/watch?v=FZKRsD9FqU4



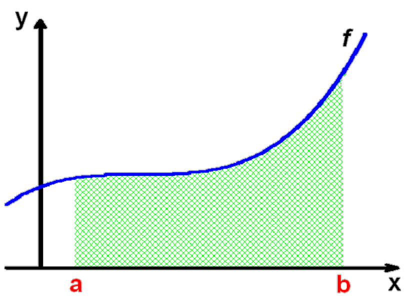
The Definite Integral is the Area of a Region

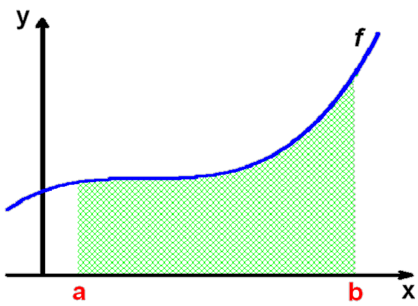
If *f* is continuous and nonnegative on the closed interval [a, b], then the area of the region bounded by the graph of *f*, the x-axis, and the vertical line x = a and x = b is given by

$$\lim_{\Delta x \to 0} \sum_{i=1}^{\infty} f(x_i)\Delta x_i$$

$$\int_a^b f(x)dx$$

height base

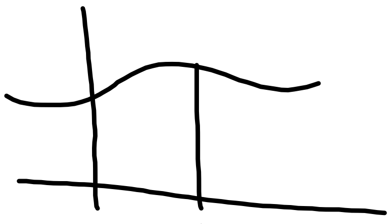




DEFINITIONS OF TWO SPECIAL DEFINITE INTEGRALS

1. If f is defined at $x = a$, then we define $\boxed{\int_a^a f(x)\,dx = 0.}$

2. If f is integrable on $[a, b]$, then we define $\boxed{\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.}$



No base

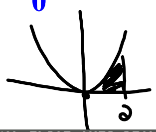
0

We can use integrals to find area under curves or to find accumulation.

Example 1

Use $\int f(x)dx$ to evaluate in the graph screen.

$$\int_0^2 x^2 dx$$



$$\int_2^0 x^2 dx$$

$$-\int_0^2 x^2 dx$$

To erase:
use **clear draw**!

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DRAW POINTS STO
1:ClrDraw
2:Line(
3:Horizontal
4:Vertical
5:Tangent(
6:DrawF
7↓Shade(
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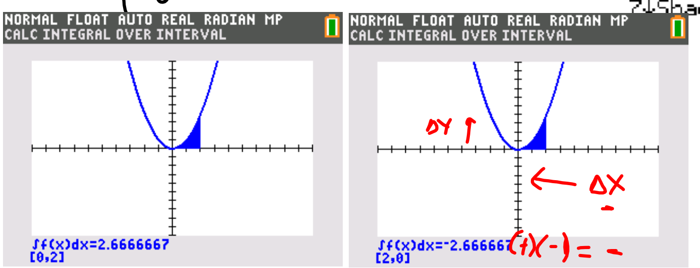
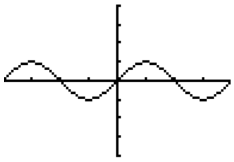


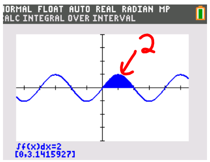
can also use math ⟶ fnInt (y_1, x, 0, 2)

Example 2

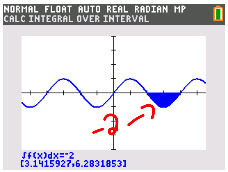
Use the calculator to evaluate in the graph screen...graph each one.



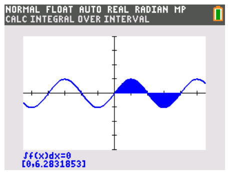
$\int_0^{\pi} \sin x\,dx$



$\int_{\pi}^{2\pi} \sin x\,dx$

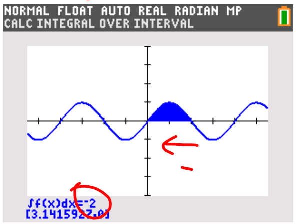


$\int_0^{2\pi} \sin x\,dx$

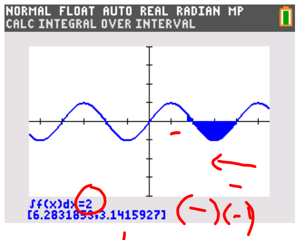


$\int_{\pi}^{0} \sin x\,dx$

$-\int_0^{\pi} \sin x\,dx$



$\int_{2\pi}^{\pi} \sin x\,dx$



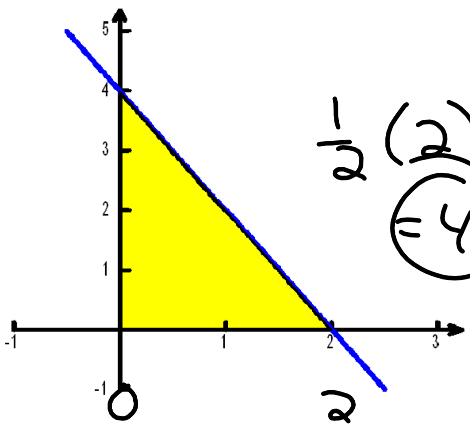
$(-)(-)$

$+$

Set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

$f(x)=4-2x$

$\int_0^2 (4-2x)\,dx$



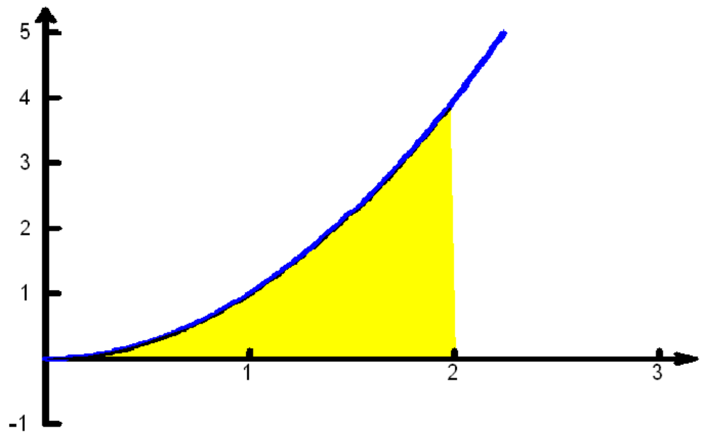
$\frac{1}{2}(2)4$

$=4$

Set up a definite integral that yields the area of the region. (Do not evaluate the integral.)

$f(x)=x^2$

$\int_0^2 x^2\,dx$



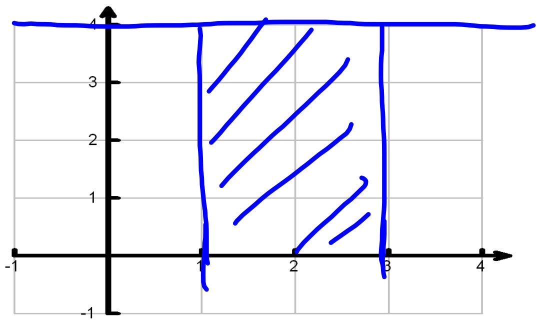
Sketch the region corresponding to each definite integral. Then evaluate each integral using a <u>geometric formula.</u>

$f(x)=4$

a. $\int_1^3 4dx$

$2(4)$

8



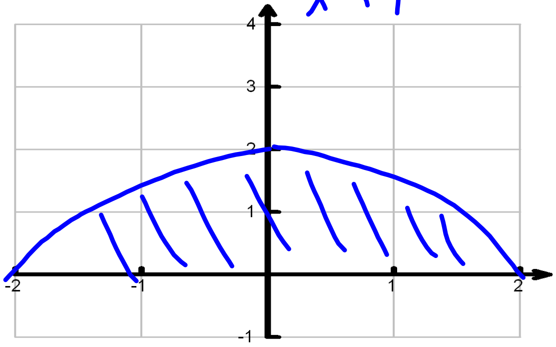
Sketch the region corresponding to each definite integral. Then evaluate each integral using a geometric formula.

$$\int_{-2}^{2} \sqrt{4-x^2}\,dx$$

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$



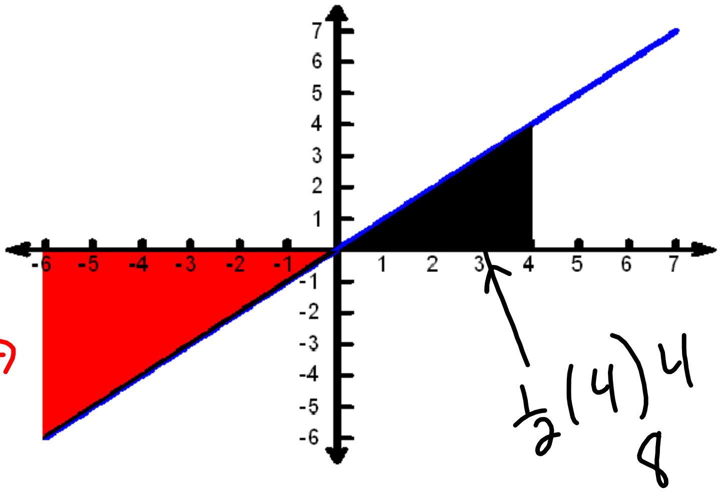
$\frac{\pi r^2}{2}$

$\frac{\pi (2)^2}{2}$

2π

Evaluate

$$\int_{-6}^{4} x\,dx$$



$\frac{1}{2}(6)(-6)$

-18

$\frac{1}{2}(4)4$

8

$+8$

-10

THEOREM 4.7 PROPERTIES OF DEFINITE INTEGRALS

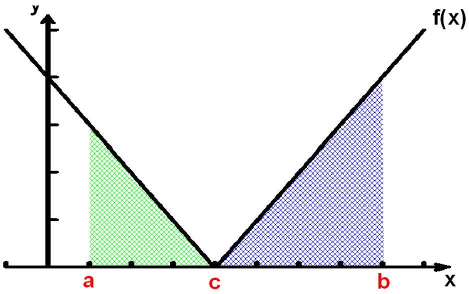
If f and g are integrable on $[a, b]$ and k is a constant, then the functions kf and $f \pm g$ are integrable on $[a, b]$, and

1. $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
2. $\displaystyle\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$

THEOREM 4.6 ADDITIVE INTERVAL PROPERTY

If f is integrable on the three closed intervals determined by a, b, and c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$



Evaluate the integrals using the following values:

$\int_1^3 x^2dx = 15$ $\int_1^3 xdx = 4$ $\int_1^3 dx = 2$

1. $\int_1^3 5xdx =$

$5\int_1^3 xdx$

$5\cdot 4 = 20$

2. $\int_3^3 x^2dx =$

0

3. $\int_1^3 (x^2 + 7)dx =$

$\int_1^3 x^2dx + \int_1^3 7dx$

$15 + 7\int_1^3 dx$

$7\cdot 2$

$15 + 14 = 29$

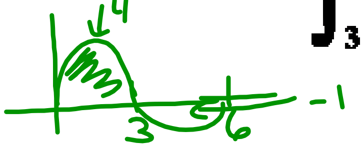
4. $\int_3^1 2x^2dx =$

$-2\int_1^3 x^2 dx$

$-2(15)$

-30

Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, evaluate



a. $\int_0^6 f(x)dx = \int_0^3 f(x)dx + \int_3^6 f(x)dx$

$4 - 1$

3

b. $\int_6^3 f(x)dx$

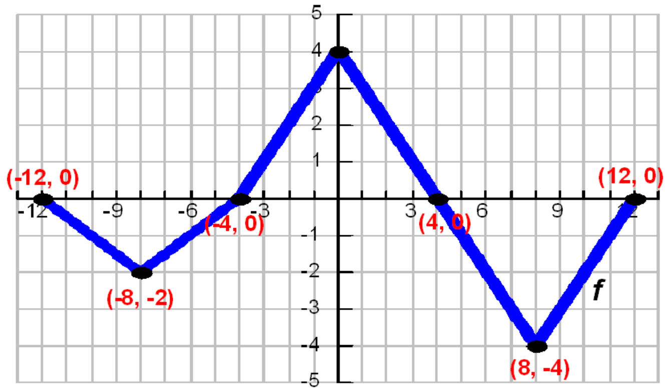
$-\int_3^6 f(x)dx$

$-(-1) = 1$

c. $\int_3^3 f(x)dx$

0

The graph of f consists of line segments, as shown in the figure. Evaluate each definite integral by using geometric formulas.



$$\int_{-12}^{-8} f(x)dx$$

$$\int_{-12}^{4} f(x)dx$$

$$\int_{-4}^{8} f(x)dx$$

Attachments

Define Integration.gsp