Use the limit definition of Area to estimate the area of...

$f(x) = 4 - x^2$, between $x = 1$ & $x = 2$ (a = 1, b = 2)

$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$

$c_i = a + \Delta x\, i$

$c_i = 1 + \frac{1}{n} i$

$f(c_i) = 4 - \left(1 + \frac{1}{n} i\right)^2$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \left(1 + \frac{1}{n} i\right)^2\right) \cdot \frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \left(1 + \frac{i}{n}\right)^2\right) \cdot \frac{1}{n}$$

$\left(1+\frac{i}{n}\right)\left(1+\frac{i}{n}\right)$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - \left(1 + \frac{2i}{n} + \frac{i^2}{n^2}\right)\right) \cdot \frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - 1 - \frac{2i}{n} - \frac{i^2}{n^2}\right) \frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} - \frac{2i}{n^2} - \frac{i^2}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 3 - \frac{2}{n^2} \sum_{i=1}^{n} i - \frac{1}{n^3} \sum_{i=1}^{n} i^2$$

$$\lim_{n\to\infty} \left(\frac{1}{n} \cdot 3n - \frac{2}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$$

$\frac{2n^3}{6n^3}$

$3 - 1 - \frac{1}{3}$

$2 - \frac{1}{3}$

$1\frac{2}{3}$

Use the limit definition of Area to estimate the area of...

$f(x) = x^3 + 2$, between $x = 0$ & $x = 1$ (a = 0, b = 1)

$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$

$c_i = a + \Delta x\, i$

$c_i = 0 + \frac{1}{n} i$

$c_i = \frac{1}{n} i$

$f(c_i) = \left(\frac{i}{n}\right)^3 + 2$

$$\lim_{n\to\infty} \sum_{x=1}^{n} f(c_i)\,\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{i}{n}\right)^3 + 2\right) \frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^3}{n^3} + 2\right) \frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} + \frac{2}{n}$$

$$\lim_{n\to\infty} \left(\frac{1}{n^4} \sum_{i=1}^{n} i^3 + \frac{1}{n} \sum_{i=1}^{n} 2\right)$$

$$\lim_{n\to\infty} \frac{1}{n^4} \frac{n^2(n+1)^2}{4} + \frac{1}{n} \cdot 2n$$

$\frac{1}{4} + 2$

$2\frac{1}{4}$

Σ DISTRIBUTES across addition and subtraction!

... but NOT multiplication and division!

Review of limits...

Find the limit of s(n) as $n \to \infty$

$$s(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$\frac{128}{6}$$

p. 260...
homewo
example

Attachments

Riemann.gsp